Trigonometry (pg. 1)

2004 FAMAT Convention

(1) $9680^\circ + n(360^\circ)$

$n = 26 \Rightarrow \underline{320^\circ}$

$\therefore$ D

(2) $MM^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & -\cos\theta\sin\theta + \cos\theta\sin\theta \\ -\cos\theta\sin\theta + \cos\theta\sin\theta & \cos^2\theta + \sin^2\theta \end{bmatrix}$

$= \underline{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$ $\therefore$ B

(3) $\tan\psi = \frac{\sqrt{3}}{3}$, $\tan\lambda = 1$ $\Rightarrow \psi = \pi/6$, $\lambda = \frac{5\pi}{4}$

$\psi + \lambda = \underline{\frac{17\pi}{12}}$ $\therefore$ C

(4) $f(x) = \sin\left(x + 2004 \cdot \frac{\pi}{2}\right)$

$= \sin(x + 1002\pi)$

$= \underline{\sin(x)}$ $\therefore$ D

(5) $y_3(x) = 10\sin(x) + 8\cos(x)$

amplitude $= \sqrt{10^2 + 8^2} \approx \underline{12.8}$ $\therefore$ A

(6) $k = \frac{\sin(52^\circ)}{\sin(31.3^\circ)} \approx \underline{1.5}$ $\therefore$ B

(7) $f(t) = \frac{(\cos^2 t + \sin^2 t) - (\cos^2 t - \sin^2 t)}{2}$

$= \underline{\sin^2(t)}$ $\therefore$ E

(8) $\sin\theta = \frac{m}{4}$ find (m, θ)

solutions: $\begin{cases} (0,0);\ (0,180^\circ);\ (1,166^\circ); \\ (1,14^\circ);\ (2,30^\circ);\ (2,150^\circ); \\ (3,49^\circ);\ (3,131^\circ);\ (4,90^\circ). \end{cases}$

$\Rightarrow \underline{9 \text{ solutions}}$ $\therefore$ C

(9) $\sin(3\theta) = \sin(\theta + 2\theta)$

$= \sin\theta\cos 2\theta + \sin 2\theta\cos\theta$

$= \sin\theta(\cos^2\theta - \sin^2\theta) + 2\sin\theta\cos^2\theta$

$= \sin\theta(1 - 2\sin^2\theta) + 2\sin\theta(1 - \cos^2\theta)$

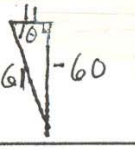
$= \underline{3\sin\theta - 4\sin^3\theta}$

$\Rightarrow A + B = 3 - 4 = \underline{-1}$

$\therefore$ C

(10) $\cos\theta = \frac{11}{61} \Rightarrow \theta \approx -79.61^\circ$

$\therefore$ B $\Rightarrow \sin\theta = \underline{-\frac{60}{61}}$



(11) $\left[-2(\cos 216^\circ + i\sin 216^\circ)\right]^5$

$= (-2)^5[\cos 1080^\circ + i\sin 1080^\circ]$

$= -32(1 + i \cdot 0) = \underline{-32}$

$\therefore$ B

(12) $\csc 2\theta = \frac{1}{2\sin\theta\cos\theta}$

$= \frac{f^2 + e^2}{2fe} = \frac{c^2}{2ab} = \underline{\frac{h^2}{2fd}} = \frac{i^2}{2ge}$

$\therefore$ B

(13) $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

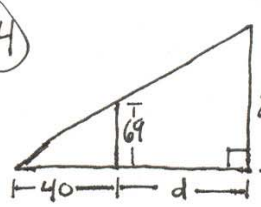
$\tan 2\theta = \frac{B}{A - C} = \frac{2}{2} = 1$

$\Rightarrow \theta = \underline{22.5^\circ}$

$\therefore$ D

$A = 1, B = 2, C = -1, D = 8, E = 1, F = 0$

(14)

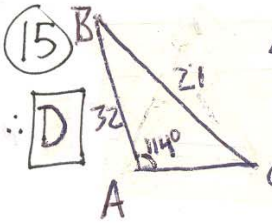


$\frac{69}{40} = \frac{240}{40 + d}$

$\Rightarrow d \approx \underline{99 \text{ inches}}$

$\therefore$ A

(15)



$\angle A = 114^\circ$, $a = 21$.

neither b nor c can be greater than a. $\Rightarrow \underline{0 \text{ solutions}}$

$\therefore$ D

Trigonometry (pg. 2)

2004 FAMAT Convention

(16) period $= \frac{2\pi}{6} = \frac{\pi}{3}$

frequency $= \frac{1}{\text{period}} = \frac{3}{\pi}$

$\therefore$ A

(17) $1 + \cos 60° + \cos^2 60° + \cos^3 60° + \dots$

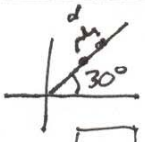
$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$

$= \frac{1}{1 - \frac{1}{2}} = 2$ $\therefore$ C

(18) $(8, 30°) \Rightarrow (4\sqrt{3}, 4)$

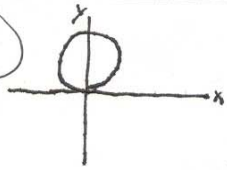
$(-6, -150°) \Rightarrow (3\sqrt{3}, 3)$

$d = 2$



$\therefore$ B

(19)



$r = \sin\theta$

$r^2 = r\sin\theta$

$x^2 + y^2 = y$

$x^2 + y^2 - y = 0$

$\therefore$ C

(20) $a^2 + 4b^2 = 62^2$

$a^2 + 9b^2 = 63^2$

$5b^2 = 63^2 - 62^2$

$b = 5$ $\therefore$ E

(21) $30°21'18''$

$= 30 + 21/60 + 18/3600$ $\therefore$ E

$= 30.355° \left(\frac{\pi \text{ rad}}{180°}\right) \approx 0.5298 \text{ rad}$

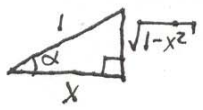
(22) Since E & O are arbitrary, let

$E = f(x) = x^2$, $O = g(x) = x^3$.

$H(x) = E/O = x^2/x^3 = 1/x$. $\therefore$ B

$H(x) = -H(-x) \Rightarrow E/O$ is odd

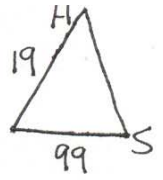
(23)



$\cos(\alpha+\beta) = \cos\alpha\sin\beta - \cos\beta\sin\alpha$

$= x\sqrt{1-4x^2} - 2x\sqrt{1-x^2}$ $\therefore$ D

(24)

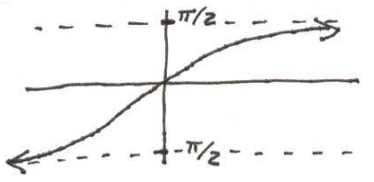


$\therefore$ C

area $\Delta LHS = \frac{1}{2}(19)(99)\sin(\angle L)$

$= 470.25$

(25)



$y = \text{Tan}^{-1}(x)$

Range:

$-\pi/2 < y < \pi/2$

$\therefore$ A

(26) $1 + \cot^2\left(\frac{45}{2}\right) = \csc^2\left(\frac{45}{2}\right)$

$= \left[\sin^2\left(\frac{45}{2}\right)\right]^{-1} = \left[\frac{1-\cos 45°}{2}\right]^{-1}$

$= \frac{2}{1 - \sqrt{2}/2} = \frac{4}{2-\sqrt{2}} \cdot \frac{2+\sqrt{2}}{2+\sqrt{2}} = 4 + 2\sqrt{2}$

$\therefore$ E

(27) $\left(\frac{\sin 30°}{\sin 60°}\right)\left(\frac{\cot 45°}{\cot 225°}\right)\left(\frac{\sec 60°}{\sec 300°}\right)\left(\frac{\csc 30°}{\csc 150°}\right)\left(\frac{\cos 120°}{\cos 240°}\right)\left(\frac{\tan 2}{\tan 6}\right)$

$= \frac{\sin 30°}{\sin 60°} \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = \frac{1/2}{\sqrt{3}/2} = \frac{\sqrt{3}}{3}$

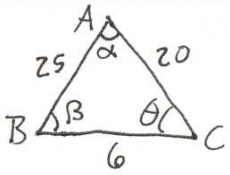
$\therefore$ E

(28) $\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \cdot \|\vec{v}\|} = \frac{-6678}{\sqrt{6517} \cdot \sqrt{7066}} \approx -0.9841$

$\Rightarrow \theta \approx 170°$

$\therefore$ C

(29)



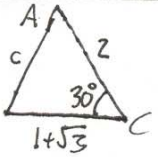
$\frac{\sin\theta}{25} = \frac{\sin\alpha}{6} = \frac{\sin\beta}{20}$

$\Rightarrow \sin\alpha + \sin\beta = \frac{6}{25}\sin\theta + \frac{20}{25}\sin\theta$

$= \frac{26}{25}\sin\theta$

$\therefore$ A

(30)



$c^2 = a^2 + b^2 - 2ab\cos\theta$

$= 4 + (1+\sqrt{3})^2 - 2(2)(1+\sqrt{3})\cos 30$

$= 2 \Rightarrow c = \sqrt{2}$

$\therefore$ E