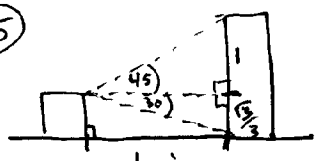
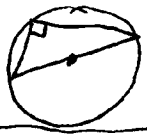
(15) 

$1+\frac{\sqrt{3}}{3}$ [D]

(16)  [A]

(17)

	1 1	
2	2 3	3
15	5 8	16
8	1 1	5
25		24

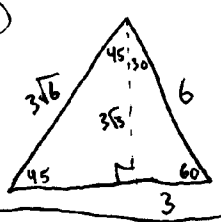
$\frac{25-24}{2} = \frac{1}{2}$. [B]

(18) $C = \sqrt{(d+e)e}$. [C]

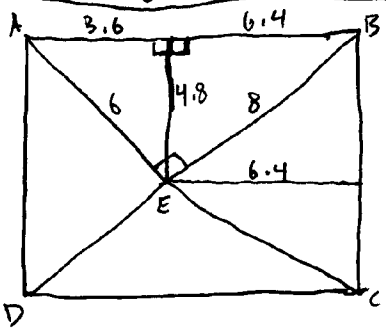
(19) Let BC = X. Using the properties of 45-45-90 triangles, we can work our way around the figure and find $AC = X(\sqrt{2})^8 = 16X$.

So, $AB = 16x - x = 15x = 30$

$x = 2$. [A]

(20) 

$3\sqrt{6}$ [C]

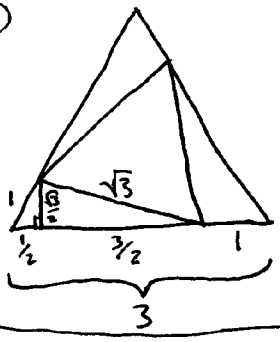
(21) 

Area of $\triangle BCE = \frac{1}{2}(BC)(6.4)$

$16 = \frac{1}{2} BC \cdot 6.4$

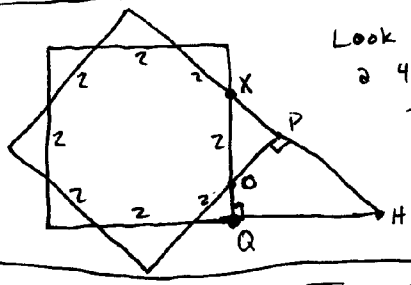
$BC = 5$

$(BC)(AB) = 5 \cdot 10 = 50$. [A]

(22) 

Area $= \frac{3^2\sqrt{3}}{4} = \frac{9\sqrt{3}}{4}$. [B]

(23) Each small triangle has equal area and equal height, so they must all also have equal bases: 4. [D]

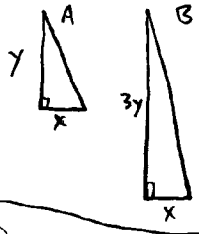
(24) 

Look at triangle XQH. It is a 45-45-90 triangle with leg $XQ = 2+\sqrt{2}$. Area of $XQH = 3+2\sqrt{2}$.

Area of QOPH = Area of XQH − Area of XPO =

$3+2\sqrt{2} - 1 = 2+2\sqrt{2}$. [A]

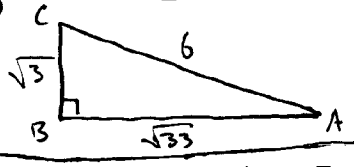
(25) $UX = \frac{1}{2} SQ = \frac{\sqrt{41}}{2}$. [D]

(26) 

Area of $A = \frac{1}{2}xy$

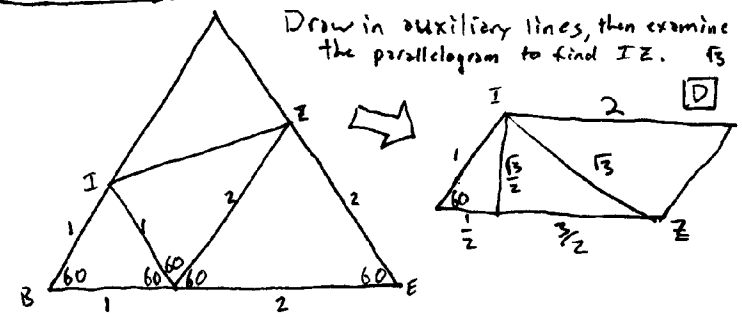
Area of $B = \frac{1}{2}3xy$

ratio $= \frac{\frac{1}{2}xy}{\frac{1}{2}3xy} = \frac{1}{3}$ [B]

(27) 

$(\sin A)^2 + (\cos A)^2 =$

$\left(\frac{\sqrt{3}}{6}\right)^2 + \left(\frac{\sqrt{33}}{6}\right)^2 = 1$. [B]

(28) 

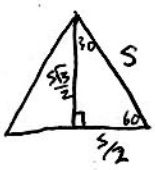
Draw in auxiliary lines, then examine the parallelogram to find IZ.

[D]

(29) $\frac{AB}{BD} = \frac{AC}{CD}$, so AD must be an angle bisector. 1:1. [E]

(30) 4 is too short, 24 is too long, 14 is just right. [B]

TRIANGLES THETA State 2009 Solutions

(1) 

$A = \frac{1}{2}bh$

$= \frac{1}{2} s \cdot \frac{s\sqrt{3}}{2} = \frac{s^2\sqrt{3}}{4}$.

C

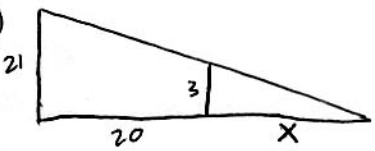
(2) $R = \frac{abc}{4A} = \frac{5 \cdot 12 \cdot 13}{4 \cdot (30)} = 6.5$ **B**

Or, note that since the triangle is right, the radius is half the hypotenuse.

(3) $A = \frac{p}{2} \cdot r$ $r = 4$. $A = 12$

$12 = \frac{p}{2} \cdot 4$

$p = 6$. **A**

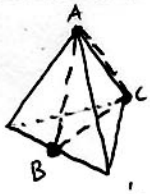
(4) 

$\frac{x}{3} = \frac{x+20}{21}$, $x = \frac{10}{3}$ **C**

(5) Total lengths $= L = (2+\sqrt{2}) + \frac{1}{2}(2+\sqrt{2}) + \frac{1}{4}(2+\sqrt{2}) + \cdots$

$2L = 2(2+\sqrt{2}) + (2+\sqrt{2}) + \frac{1}{2}(2+\sqrt{2}) + \cdots$

$L = 2L - L = 2(2+\sqrt{2}) = 4+2\sqrt{2}$. **C**

(6) 

$\triangle ABC$ is the cross section.

$AB = CB$, but AC is not. [AC = one edge of the tetrahedron. $AB = CB = \left(\frac{\sqrt{3}}{2}\right) \cdot AC$.]

$\therefore$ Isosceles only. **B**

(7) 3-4-5 These are the only ones.

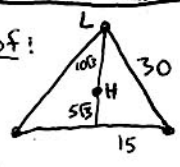
5-12-13. [After 12 and 13, the difference of any 2 consecutive squares is greater than 5^2.] **B**

(8) $\left(\frac{-5+4+7}{3}, \frac{0+1+2}{3}\right) = (2,1)$.

D

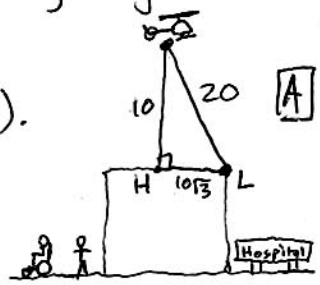
(9) Definitions. **D**

(10) The roof:



H = spot helicopter is above, (also the center of the roof).

L = signal light.



A

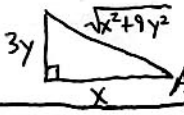
(11) No triangle can be more than one of the following: right, acute, obtuse.

So, choose: one right,

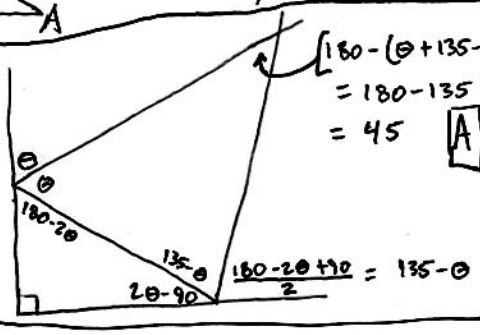
one obtuse-scalene, and

one equilateral-isosceles-acute.

This makes 3 triangles. **B**

(12) 

$\tan A = \frac{3y}{x}$. **B**

(13) 

$[180-(\theta+135-\theta)]$

$= 180-135$

$= 45$ **A**

$\frac{180-2\theta+90}{2} = 135-\theta$

(14) Max Pieces using n-cuts:

$M_n = M_{n-1} + n$.

$M_4 = 11$.

# of triangles:

$T_n = n-2$ for $n \geq 2$.

(With each new cut, every existing triangle becomes a quadrilateral and a smaller triangle, and one new triangle is made as well. This can be seen by drawing the correct cuts in order and counting the number of triangles each time.)

$T_4 = 4-2 = 2$. **B**