Vector and Multivariate Calculus

For all questions, answer E. NOTA means "none of the above answers is correct"

1 If $v = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, calculate $\|v\|$.

A. 0 B. $2\sqrt{2}$ C. $\sqrt{22}$ D. 22 E. NOTA

2 If $v = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $w = i + j + 3k$, calculate $\langle v, w \rangle$

A. 14 B. 15 C. 16 D. 17 E. NOTA

3 If $v = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $w = i + j + 3k$, calculate $v \times w$.

A. $4i + 3j + 6k$ B. $3i - 6i + k$ C. $3i + 2j + 9k$ D. 14 E. NOTA

4 A geometric interpretation of $\|v \times w\|$ is

A. The projection of *v* onto *w*
B. The magnitude of the projection of *v* onto *w*
C. The area of the parallelogram formed by *v* and *w*
D. The volume of the tetrahedron formed by *v*, *w* and the *x*-*y* plane
E. NOTA

5 The absolute value of the triple scalar product of vectors v_1, v_2 and v_3 can be interpreted geometrically as

A. The vector in the direction of v_3 with magnitude $\|v_1\| \cdot \|v_2\| \cdot \|v_3\|$
B. The volume of the tetrahedron formed by the three vectors.
C. The sum of the magnitudes of the vectors.
D. The volume of the parallelepiped formed by the three vectors.
E. NOTA

6 The dot product of vectors v_1 and v_2 can be interpreted geometrically as

A. The vector in the direction of v_2 with a magnitude of $\|v_1\| \cdot \|v_2\|$.
B. The product of the length of the projection of v_1 onto v_2 and the length of v_2.
C. The angle between the two vectors.
D. The product of the length of v_1 and the length of v_2.
E. NOTA

7 What is the intersection point of the line that passes through points $P = (-3, -1, 2)$ and $Q = (5, 8, 4)$ and the *x*-*y* plane?

A. (–11,-10,0)
B. $(8,9,2)t + (5,8,4)$
C. $\left(1, \frac{7}{2}, -1\right)$
D. $\left(1, \frac{7}{2}, 0\right)$
E. NOTA

8 What is the equation of the plane that contains the points?
$A=(4,1,2), B=(1,5,4),$ and $C=(-3,2,6)$

A. $3x+2y-3z=8$ B. $4x+5y+z=33$ C. $14x-2y+25z=0$
D. $14x-2y+25z=104$ E. NOTA

9 Find the curl of the vector field $\mathbf{F}(x,y,z)=xyz\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ at the point (1,2,1).

A. $-2\mathbf{j}-\mathbf{k}$ B. $-2\mathbf{j}-2\mathbf{k}$ C. $2\mathbf{j}-\mathbf{k}$ D. $\mathbf{i}+\mathbf{j}+\mathbf{k}$ E. NOTA

10 For $f(x,y)=y\log(4x+y^2)$, calculate $f_{xy}-f_{yx}$.

A. $\dfrac{4y}{4x+y^2}$ B. $\dfrac{2y^2}{4x+y^2}$ C. $\dfrac{4y-2y^2}{4x+y^2}$ D. $\dfrac{4(4x-y^2)}{(4x+y^2)^2}$ E. NOTA

11 Which of the following points is NOT on the plane tangent at $P=(1,1,3)$ to the surface $z=2x^2+y^2$?

A. $(1,1,3)$ B. $(2,3,11)$ C. $(2,-3,-1)$ D. $(3,-2,7)$ E. NOTA

12 Find the divergence at (2, 1, -1) for the vector field.
$\mathbf{F}(x,y,z)=x^3y^2z\mathbf{i}+x^2z\mathbf{j}+x^2y\mathbf{k}$

A. –20 B. –15 C. –12 D. –8 E. NOTA

13 Let $w=f(x,y,z)=yz^2-x$, where *x*,*y*, and z are the following functions of *t*:
$x(t)=t^2$
$y(t)=2t-3$.
$z(t)=1-t$

Find $\dfrac{dw}{dt}$ at *t=1*.

A. –3 B. –2 C. 0 D. Does Not Exist E. NOTA

14 Let $f(x,y)=\sqrt{81-x^2-y^2}$. Find the rate of change of *f* as we move from the point $P=(1,4)$ in the direction toward the point $Q=(4,8)$.

A. $-\dfrac{19}{40}$ B. $-\dfrac{2}{5}$ C. $\dfrac{2}{5}$ D. $\dfrac{19}{40}$ E. NOTA

15 Which of the following is a critical point of the function: $f(x,y)=x^2+y^2-6xy$

A. (0, 0) B. (3, 1) C. (1, 3) D. All of these are critical points E. NOTA

16 $\int_0^4 \int_y^2 \left(y^2 - xy\right) dxdy$

A. $-\frac{16}{3}$ B. 0 C. $\frac{11}{3}$ D. $2y^2 - 2y - \frac{y^3}{2}$ E. NOTA

17 Which of the following vectors is *not* orthogonal to $v = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$?

A. **i – j – k** B. **i + 2j** C. 3**i**-2**k** D. **i** + 3**j** - **k** E. NOTA

18 What is the area of the triangle whose vertices are (0,0,1), (0,2,0), and (3,0,0)?

A. 8/3 B. 7/2 C. 3 D. 6 E. NOTA

19 Find the perpendicular distance from the origin to the plane $x + 2y + 2z = 6$.

A. 1 B. 4/3 C. 2 D. 3 E. NOTA

20 If the variables *P, V* and *T* are related by the equation $PV = nRT$, where *n* and *R* are constants, simplify the expression

$$\frac{\partial V}{\partial T} \cdot \frac{\partial T}{\partial P} \cdot \frac{\partial P}{\partial V}$$

A. –1 B. 1 C. $-nR$ D. nR E. NOTA

21 The equation $x^3z^5 - y^2z^3 - 3xy = 1$ defines an implicit function $z = f(x, y)$. What is the value of $\frac{\partial f}{\partial y}$ at the point $(x, y) = (-1, 1)$?

A. –8 B. –1 C. $-\frac{1}{8}$ D. $\frac{1}{8}$ E. NOTA

22 A vertical fence is constructed whose base is the curve $y = x\sqrt{x}$, from $(0,0)$ to $(1,1)$, and whose height above each point (x, y) along the curve is $x^3 - y^2 + 27$. Find the area of the fence.

A. $\frac{1}{9}\left(5\sqrt{5} - 2\right)$ B. $5\sqrt{13} - 6$ C. $9\sqrt{3}$ D. $13\sqrt{13} - 8$ E. NOTA

23 Evaluate

$$\int_0^4 \int_0^1 \int_0^x dydxdz$$

A. 0 B. 1 C. 2 D. 4 E. NOTA

24 Find the length of the vector $\langle 2, 2, 3 \rangle$.

A. $\sqrt{13}$ B. $\sqrt{14}$ C. $\sqrt{15}$ D. $\sqrt{17}$ E. NOTA

25 Find a unit vector that has the same direction as the vector $\langle 2,2,1\rangle$.

A. $(1,1,1)$ B. $\left(\frac{1}{2},\frac{1}{2},1\right)$ C. $\left(\frac{2}{3},\frac{2}{3},\frac{1}{3}\right)$ D. $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$ E. NOTA

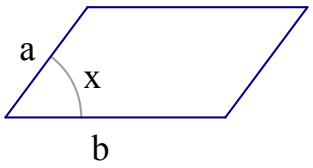
26 Find the minimum value of the function $f(x,y)=x^2+y^2+2x-2y$.

A. f has no minimum value B. -8 C. -2 D. 4 E. NOTA

27 What is the norm of the partition of the region $R=\{(x,y)\,|\,0\le x\le 1, 0\le y\le 1\}$ if it is partitioned by the lines $x=\frac{1}{4}, x=\frac{1}{2}, x=\frac{3}{4}$ and $y=\frac{1}{2}$?

A. $\frac{\sqrt{3}}{4}$ B. $\frac{\sqrt{5}}{4}$ C. $\frac{1}{2}$ D. $\frac{1}{4}$ E. NOTA

28



If the area of the pictured parallelogram is A, find the rate of change of A with respect to side a if $a=10$, $b=20$ and angle $x=\frac{\pi}{6}$.

A. 10 B. 100 C. $100\sqrt{3}$ D. $\frac{100\pi}{3}$ E. NOTA

29 Find $\frac{\partial z}{\partial x}$ given $3x^2z-x^2y^2+2z^3+3yz-7=0$ at $(1,1,1)$.

A. $-\frac{2}{3}$ B. $-\frac{1}{3}$ C. 0 D. Does Not Exist E. NOTA

30 Find ∇f at the point (1,2) if $f(x,y)=y\ln x+xy^2$.

A. 0 B. (6, 4) C. (4, 6) D. $\sqrt{52}$ E. NOTA