

Theta Topic Test - Circumference, Perimeter, Area, and Volume  
FAMAT State Convention 2003

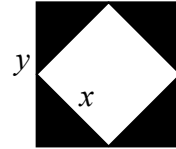
For all questions, answer "E) NOTA" means none of the above answers is correct.  
The figures in this test are not drawn to scale.

- 1) Find the area between the graphs  $y = 0$ ,  $x + y = 7$ , and  $x - y = -3$  on the Cartesian plane.

A) 12.5      B) 25      C) 50      D) 62.5      E) NOTA

- 2) The figure below shows a white square of side length  $x$  on top of a black square of side length  $y$ . Find the area of the black region shown in terms of  $x$ .

A)  $\frac{x}{2}$       B)  $x$       C)  $2x$       D)  $x^2$       E) NOTA



- 3) A certain triangle has sides of length 7, 8, and 9. What is the area of the smallest triangle that is similar to the original and has a side of length 1?

A)  $\frac{4\sqrt{5}}{27}$       B)  $\frac{12\sqrt{5}}{49}$       C)  $\frac{24}{7}$       D)  $\frac{8}{3}$       E) NOTA

- 4) A square has diagonal of length  $7\sqrt{2}$ . Find the ratio of its perimeter to area.

A)  $\frac{1}{7}$       B)  $\frac{7}{7\sqrt{2}}$       C)  $\frac{14}{49}$       D)  $\frac{4}{7}$       E) NOTA

- 5) An isosceles right triangle which has legs of length  $x$  is inscribed in a circle such that the hypotenuse of the triangle serves as a diameter of the circle. The area of the circle is  $16\pi$ . Find  $x$ .

A) 4      B) 5      C)  $4\sqrt{2}$       D)  $8\sqrt{2}$       E) NOTA

- 6) In a regular polygon, each interior angle is seven times the measure of its corresponding exterior angle. How many sides does this polygon have?

A) 10      B) 12      C) 16      D) 18      E) NOTA

- 7) A Gamma sequence is defined such that  $a_n = (1 + n)^2 - 1$ . An equilateral triangle has side length equal to the 2nd term in the sequence. Find the area of this triangle.

A)  $8\sqrt{3}$       B)  $16\sqrt{3}$       C)  $64\sqrt{3}$       D)  $128\sqrt{3}$       E) NOTA

- 8) Find the area of a triangle with vertices at points A, B, and C. The points are A(5,-7), B(1,0) and C(4,10). (Note: Round your answer to the nearest tenth.)

A) 25.5      B) 30.5      C) 51.0      D) 61.0      E) NOTA

9) The endpoints of a line segment are given by  $(f(g(2)), g(f(3)))$  and  $(f(g(0)), g(f(1)))$ . If  $f(x) = 4x - 2$  and  $g(x) = x + 4$ , find an equation of the line connecting these two points in Standard Form.

- A)  $x - y = 8$       C)  $x - y = -8$       E) NOTA  
 B)  $x + y = 8$       D)  $x + y = -8$

10) Find the area of the circle with the following equation:  $x^2 + y^2 - 8x + 14y + 16 = 0$ .

- A)  $36\pi$       B)  $49\pi$       C)  $64\pi$       D)  $81\pi$       E) NOTA

11) A circle is such that 65% of the value of the area of the circle is equal to the value of its circumference. Find the area of this circle, to the nearest hundredth.

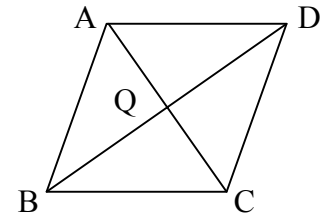
- A) .33      B) 5.31      C) 27.94      D) 29.74      E) NOTA

12) Two chords, AB and CD intersect inside a circle at point E (not the center). You're told that  $DE = x - 3$ ,  $BE = x - 4$ ,  $CE = x$ ,  $AE = x - 1$ . A regular hexagon has side length of the same  $x$ . Find the perimeter of this hexagon.

- A) 3      B) 6      C) 12      D) 18      E) NOTA

13) In the rhombus,  $\overline{AC}$  and  $\overline{DB}$  are diagonals intersecting at Q. If  $AQ = 4$  and  $QB = 5$ , find the area of ABCD.

- A)  $\sqrt{41}$       B) 18      C) 20      D) 40      E) NOTA



14) A circle has center at  $(2,4)$  and the point  $(x_0, y_0)$  lies on the circle where  $x_0$  and  $y_0$  represent the intersection of the lines  $3x + 2y - 7 = 0$  and  $4x - 5y + 2 = 0$ . Find the area of this circle to the nearest tenth.

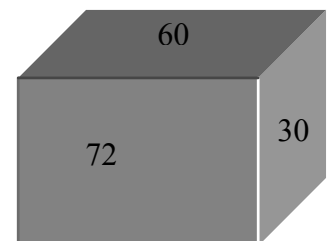
- A) 8.5      B) 10.7      C) 21.3      D) 42.6      E) NOTA

15) A pendulum seven feet long “sweeps” an arc of  $17.3^\circ$  on both sides of its center position. Find the area of the sector “swept” out by this pendulum (as it swings in a plane) to the nearest tenth.

- A) 2.1      B) 7.4      C) 14.8      D) 46.4      E) NOTA

16) You know the surface area of three faces of a box (right rectangular prism) to be 72, 60, 30. These represent the front, top, and side of the box respectively as shown below. Find the volume of this box. (Hint: the length of the box sides are integers.)

- A) 162      B) 360      C) 529      D) 648      E) NOTA

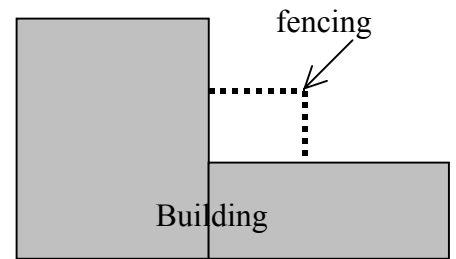


17) A square and an equilateral triangle both have area of  $16\sqrt{3}$ . Find the ratio of the length of the square side to the length of the triangle side. (Round your answer to the nearest hundredth).

- A) 0.25      B) 0.66      C) 1.52      D) 4      E) NOTA

18) Cleedus the farmer is building a fence for his pigs. He plans on using the two sides of a building and using two sides of fence for a rectangular pen. He only has \$500 to spend on the fencing and each foot of fence costs \$10. He needs to house as many pigs as possible (i.e. the area enclosed should be maximized.) Given that he can only use integer feet lengths of fence, what is the largest fenced area (in square feet) he can have?

- A) 50  
B) 600  
C) 625  
D) 2500  
E) NOTA



19) Two triangles, A and B, are similar. Triangle A has area of 16, triangle B has area 9. Given that the perimeter of triangle A is  $x$ , what is the perimeter of triangle B (in terms of  $x$ ).

- A)  $\frac{9}{16}x$       B)  $\frac{3}{4}x$       C)  $x$       D)  $\frac{4}{3}x$       E) NOTA

20) An isosceles right triangle has legs of length  $\pi$ . A related circle has a circumference equal to the length of the hypotenuse of the isosceles right triangle. Find the volume encapsulated if this circle is rotated about a diameter sufficiently to create a sphere.

- A)  $\frac{\pi\sqrt{2}}{6}$       B)  $\frac{\pi\sqrt{2}}{3}$       C)  $\frac{\pi\sqrt{2}}{2}$       D)  $\pi\sqrt{2}$       E) NOTA

21) Conan is considering making a dartboard. He can't decide which design to use for his board. His options are shown below. Board A is formed with concentric circles of radii 1 (bullseye), 2, 3, 4, and 5 respectively. Board B is formed by concentric squares of semi-perimeters 4, 6, 8, and 10 respectively. Which of Conan's two options has the largest shaded areas?

- A) Board A  
B) Board B  
C) The shaded areas are equal  
D) Not enough information is given  
E) NOTA



22) Two similar circles have circumferences of 26 and 16 respectively. Find the ratio of radii (smallest:largest).

- A)  $\frac{8}{13}$       B)  $\frac{2\sqrt{26}}{13}$       C) 64:169      D)  $\frac{\sqrt{26}}{4}$       E) NOTA

23) A trapezoid has bases of length 10 and 20 with height of 8. Find the area of the trapezoid.

- A) 40      B) 60      C) 80      D) 120      E) NOTA

24) A circle has diameter of 10. Its area increases such that the area of the new circle is 25 times the area of the original. Find the new radius.

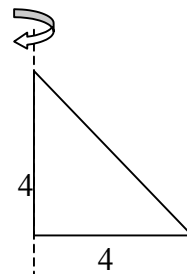
- A)  $\sqrt{5}$                       B) 10                      C) 25                      D) 50                      E) NOTA

25) The midpoints of the sides of an equilateral triangle, A, are connected to form a new triangle, B. Then the midpoints of triangle B are connected to form a new triangle, C. Find the ratio of the area of triangle C to triangle A.

- A)  $\frac{1}{4}$                       B)  $\frac{1}{8}$                       C)  $\frac{1}{12}$                       D)  $\frac{1}{16}$                       E) NOTA

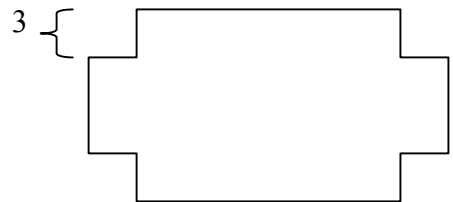
26) A right isosceles triangle is rotated about one leg (shown as a dotted line) to form a 3-D figure. Find the lateral surface area created when this rotates to form a cone.

- A)  $16\pi$                       C)  $16\pi\sqrt{2}$                       E) NOTA  
 B)  $8\pi\sqrt{2}$                       D)  $32\pi\sqrt{2}$



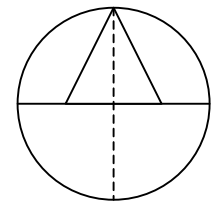
27) A piece of 11 x 14 inch paper is cut as shown below so as to form a box (without a top) when folded. If squares of side length “x” are cut out, find the volume of the box if  $x = 3$  inches.

- A) 120                      C) 264                      E) NOTA  
 B) 192                      D) 462



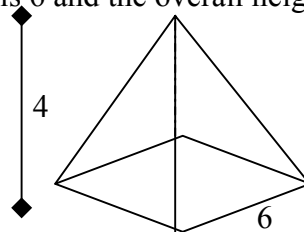
28) Two vertices of an equilateral triangle lie on a diameter of a circle and the third vertex lies on the circle whose area is  $121\pi$  as shown below. The dotted line (a diameter) is the perpendicular bisector of the this diameter (this diameter is shown as a solid line) and intersects the triangle at the vertex tangent to the circle. Find the area of the triangle (round your answer to the nearest tenth).

- A) 1.5                      C) 47.5                      E) NOTA  
 B) 17.5                      D) 69.9



29) A regular pyramid is shown below. The side length of the square base is 6 and the overall height is 4. Find the lateral surface area.

- A) 48  
 B) 60  
 C) 96  
 D) 120  
 E) NOTA



30) A square is inscribed in a circle of radius 5. If another circle is inscribed inside the square, find the circumference of the smaller circle.

- A)  $\pi\sqrt{2}$                       B)  $3\pi\sqrt{2}$                       C)  $5\pi\sqrt{2}$                       D)  $7\pi\sqrt{2}$                       E) NOTA