

**2003 FAMAT STATE CONVENTION
THETA BOWL QUESTIONS**

QUESTION 1

$F(x) = 2x + 1$ and $G(x) = x^3 - 2$. Evaluate $F(1) + G(2) + F(G(3)) + F^{-1}(4)$.

QUESTION 2

The sides of a triangle have lengths x , $\sqrt{5x}$, and $\sqrt{6}$. What is the sum of all values of x which make this a right triangle?

QUESTION 3

The first 6 terms of the binomial expansion of $(1+2)^{\frac{1}{2}}$ sum to $\frac{A}{B}$, a positive fraction in lowest terms. What is $A + B$?

QUESTION 4

Given the equation $32x^4 - 72x^3 - 564x^2 + 730x + 1575 = 0$

Let A = the sum of the roots

Let B = the product of the roots

Let C = the sum of the reciprocals of the roots

Let D = the sum of the squares of the roots

Give the value of $\frac{-ACD}{B}$ as a fraction in lowest terms.

QUESTION 5

$$A = \sqrt{552 + \sqrt{552 + \sqrt{552 + \dots}}}$$

$$B = \sum_{n=1}^{15} n^3$$

$$C = \frac{1}{5} + \frac{2}{25} + \frac{3}{125} + \frac{4}{625} + \dots$$

Find $\frac{2BC}{A}$.

QUESTION 6

A line with a slope of 3 is tangent to the parabola $y = x^2 - 4x + 6$. What is the y-intercept of this line?

QUESTION 7

A ball is dropped from a height of 420 meters. When it hits the ground, it rebounds to $\frac{5}{6}$ its original height, or 350 meters. The ball continues to rebound to $\frac{5}{6}$ its previous height after its second bounce, but from the 3rd bounce on, the ball loses some of its “bounciness” and only rebounds to one-half its previous height. What is the total number of meters that the ball travels?

QUESTION 8

Andrea has invented a new product called Andrea’s Acidic Solution, or AAS, which is a solution of acid and other chemicals. She wants her AAS to have as much acid as possible, but in her attempts to create the perfect AAS, she found that 40% or more acid will eat through its container, and she won’t be able to sell it. Right now her new batch of AAS is 100 gallons of 16.5% acid. She has an unlimited supply of acid in indestructible containers that each hold 0.12 gallons of acid. What is the number of whole containers of acid she can pour in her new batch to get the most acidic solution that will not eat through its container?

QUESTION 9

Let A = the number of digits in 4^{44}
Let B = the last digit of $7^{2003} + 8^{2003} + 9^{2003}$
Let C = the second to last digit of 11^{2003}

Find the value of $AB + C^2$.

QUESTION 10

Consider a 10 foot by 12 foot sheet of paper.
Let A = the volume of the topless box formed by cutting 2 foot by 2 foot squares from each corner and folding the sides up
Let B = the outside surface area of the same box
Find $10B - 3A$.

QUESTION 11

5 pipes are connected to a tank. Pipes 1, 2, 3, and 4 can each fill an empty tank in 1 hour, 2 hours, 3 hours, and 4 hours, respectively. Pipe 5 can drain a full tank in 30 minutes. The tank starts out empty. Pipe 1 is turned on, and after 5 minutes, pipe 2 is turned on, then after 5 more minutes, pipe 3 is turned on, then 5 minutes later, pipe 4 is turned on. Five minutes after that, all 4 pipes are shut off, and pipe 5 is turned on. How many seconds does it take pipe 5 to completely empty the tank?

QUESTION 12

What is the area of the region bounded by the graphs of $|x| + |y| \leq 6$ and $|x| + |y| \geq 1$?

QUESTION 13

Y varies directly as the square of X and inversely as the cube of Z. If $Y = 32$ when $X = 4$ and $Z = 3$, find Y when $X = 6$ and $Z = 0.5$.

QUESTION 14

Using the scoring formula $4(\text{number correct}) - (\text{number incorrect})$, what is the 6th highest possible score on a 30 question test?

QUESTION 15

Using only quarters, dimes, and nickels,

Let B = the number of different coin combinations that make \$.30 with at least one nickel

Let C = the number of different coin combinations that make \$.30 with at least one dime

Let D = the number of different coin combinations that make \$.30 with at least one
quarter

Find $B + C + D$.