

Calculus Applications

2003 FAMAT State Convention

For all questions, **NOTA** means **None Of The Aforementioned** answers is correct.

Note: when units are specified, answers may be presumed to be in the correct units.

1. Which of the following correctly represents $\int \sin(x) \cos(x) dx$?

i) $\frac{\sin^2(x)}{2}$ ii) $-\frac{\cos^2(x)}{2}$ iii) $-\frac{\cos(2x)}{4}$

- a) i and ii only b) i, ii, iii c) i only d) iii only e) NOTA

2. Newton's second law of motion states that $F = \frac{d(m(t) \cdot v(t))}{dt}$. A snowball begins at rest with negligible mass and accelerates down a slope at $1 \frac{m}{s^2}$ while gaining mass at the rate of $1 \frac{kg}{s}$. Determine the force (in Newtons) the snowball would exert if it were to hit a stationary object 10 seconds after it begins to move. (Remember $N = \frac{kg \cdot m}{s^2}$.)

- a) 10 b) 20 c) 30 d) 80 e) NOTA

3. Consider the finite area bounded by a parabola and its latus rectum. Revolving this area about the latus rectum forms a football-shaped object. If the football-shaped object is 12 in. along the axis of revolution, calculate the volume of revolution in in^3 . (Hint: It may be helpful to use the equation $y = -\frac{x^2}{12} + 3$.)

- a) $\frac{288\pi}{5}$ b) $\frac{162\pi}{5}$ c) $\frac{128\pi}{7}$ d) $\frac{64\pi}{7}$ e) NOTA

4. Evaluate $\frac{dy}{dx}$ at $x=1$ for $y = e^\pi$.

- a) e^π b) $\pi e^{\pi-1}$ c) 0 d) 1 e) NOTA

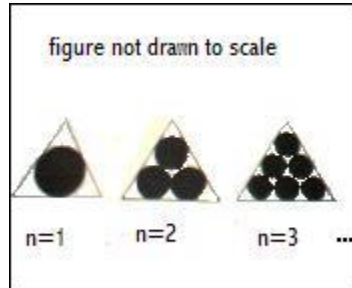
5. Given some of the following conversion factors, calculate the cost of a hamburger in dollars per *MBtu*.

$$\left(\frac{\$1.00}{\text{hamburger}} \right) = \left(\frac{2 \cdot 10^6 J}{\text{hamburger}} \right) = \left(\frac{1055 \cdot 10^6 J}{\text{MBtu}} \right) = \left(\frac{J}{N \cdot m} \right) = 1$$

- a) $2110 \cdot 10^{12}$ b) 263.75 c) 2110 d) 527.50 e) NOTA

6. Within an equilateral triangle of side length s , tangent circles are drawn of equal radius r . Determine the maximum area of the shaded region as the number of circles goes to infinity.

(Hint: $\lim_{n \rightarrow \infty} r = \frac{s}{2(n + \sqrt{3} - 1)}$ where n is the number of circles tangent to one side of the triangle.)



- a) $\frac{\pi}{8}s^2$ b) $\frac{\pi}{3}s^2$ c) $\frac{2\pi}{\sqrt{3}}s^2$ d) $\frac{\pi}{\sqrt{6}}s^2$ e) NOTA
7. Which of the following terms commonly identifies the third derivative of position with respect to time?
- a) delta b) epsilon c) jerk d) snatch e) NOTA
8. A tub of ice cream is purchased at the local grocery where it was kept cold inside a freezer at -60°F . Upon leaving the store it is boarded onto a hot bus at 80°F . 10 minutes later the ice cream's temperature is measured to be 0°F . Assuming heat transfer only occurs on the bus, how long can the ice cream remain on the bus until it completely melts at 32°F ? Use Newton's Law of Heating and round your answer to the nearest minute.
- a) 88 b) 24 c) 15 d) 19 e) NOTA
9. A normal bell curve is described by the equation $p(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}}$. A point of inflection in this curve exists one standard deviation from the mean. Calculate $|x|$ for this point.
- a) $2/3$ b) 1 c) $\frac{1}{\sqrt{2\pi}}$ d) $\frac{1}{2\pi}$ e) NOTA
10. A spring obeys Hooke's Law with a spring constant $k = 10 \frac{\text{N}}{\text{m}}$. A block comes into contact with the spring to depress it 0.5m from equilibrium. How much work (in Joules) does the block perform on the spring? (Hint: Remember $J = \text{N} \cdot \text{m}$.)
- a) $1/8$ b) 4 c) 16 d) 10 e) NOTA
11. Use one iteration of Newton's method to approximate the positive root of $y = x^2 - 5$. Use an initial guess of $x = 2.5$.
- a) 19 b) 28 c) 31 d) 45 e) NOTA

12. A balloon in the shape of a sphere is being blown up at the rate of $1 \frac{m^3}{s}$. At what rate (in $\frac{m^2}{s}$) is the surface area increasing when the radius is $0.25m$?

- a) 16π b) 8π c) 8 d) 16 e) NOTA

13. Which of the following functions satisfy the equation $y''(x) + y(x) = 0$?

- i) $y(x) = \cos(x) + \sin(x)$ ii) $y(x) = e^{-x} + e^x$ iii) $y(x) = \cos(x) + e^{-x}$ iv) $y(x) = 0$

- a) i, ii, iii, iv b) ii, iii, iv only c) i, iii only d) i, iv only e) NOTA

14. In a beam the shear force V is related to bending moment M by $V(x) = \frac{dM(x)}{dx}$. For a beam of length 6 the moment at any point x along the beam is expressed as

$$M(x) = \begin{cases} 800x - 3600, & 0 \leq x < 3 \\ 400x - 2400, & 3 \leq x \leq 6 \end{cases}$$

Calculate $V(5)$.

- a) 800 b) 400 c) 600 d) 0 e) NOTA

15. How many local maxima exist for $f(x) = \sin(x) + x$?

- a) 0 b) 1 c) ∞ d) can't be determined e) NOTA

16. An object experiencing undamped oscillation has an equation of motion $x(t) = A \cdot \cos(\omega \cdot t)$. The object's acceleration then is related to the motion as $a(t) = K \cdot x(t)$. Determine the constant of proportionality K .

- a) ω^2 b) $-\omega$ c) $A\omega^2$ d) $-A\omega$ e) NOTA

17. A damped system oscillates an object with equation of motion $x(t) = A \cdot e^{-ct} \cdot \cos(\omega t)$. Determine the amplitude of oscillation as $t \rightarrow \infty$.

- a) 0 b) $-A\omega^2$ c) $Ac^2\omega^2$ d) $-Ac^2$ e) NOTA

18. Evaluate $\lim_{x \rightarrow \infty} (\sqrt{3x^2 + 2x + 1} - \sqrt{3x^2 - x})$.

- a) 1 b) 1.5 c) $\frac{\sqrt{3}}{2}$ d) $\frac{\sqrt{6}}{3}$ e) NOTA

19. On a certain day the temperature t -hours after sunrise was $T(t) = 80 + 10 \sin\left(\frac{\pi}{12}t\right)$. Determine the average temperature (in degrees Fahrenheit) between sunrise and sunset, 12 hours later.

- a) 80 b) $80 + \frac{20}{\pi}$ c) $80 + \frac{10}{\pi}$ d) $80 + \frac{6}{\pi}$ e) NOTA

26. A search light which makes one revolution every 20 seconds is located 300 feet from a long, straight wall. How fast is the spotlight on the wall moving when the beam of light makes a 45° angle with the wall?
- a) 60π b) 120π c) 30π d) 5π e) NOTA
27. A comet enters our solar system traveling on a path in the shape of a parabola with the sun as the focus and at the origin. The equation of the directrix is found to be $y = -2.0 \times 10^8$ (miles). Find the equation of the parabola in $y = Ax^2 + Bx + C$ form, leaving each coefficient in units of 10^8 miles.
- a) $y = 0x^2 + 0.5x - 1$ b) $y = 0.25x^2 + 0x - 1$ c) $y = 0.5x^2 + 0x - 1$
d) $y = 0.25x^2 - 0.5x + 1$ e) NOTA
28. A baseball team plays in a stadium with 48,000 capacity seating. On average, 20,000 fans will pay \$20 per seat. A marketing survey shows that for every dollar the average ticket price is lowered, 3,000 more fans will attend home games. Determine to the nearest dollar, what average ticket price will maximize ticket revenue.
- a) $6\frac{2}{3}$ b) 7 c) 13 d) 27 e) NOTA
29. Electrical engineers note that the charge Q on a capacitor is related to the current I in a circuit by the differential relation $\frac{dQ(t)}{dt} = I(t)$. For $I(t) = \frac{V}{R} e^{-\frac{t}{RC}}$ where V , R , and C are constant, determine the charge stored on capacitor between $t=0$ and $t=5RC$.
- a) $CV(1 - e^{-5})$ b) $RC(e^5 - 1)$ c) $\frac{V}{R}(e^5 - 1)$ d) $\frac{C}{R}(1 - e^{-5})$ e) NOTA
30. Marshall's golden arm fires a football into the air with initial velocity $v_0 = \sqrt{190} \frac{\text{yard}}{\text{s}}$ and at an inclined angle Θ . Upon leaving his hand, the ball travels a horizontal distance according to the range equation below. What is the maximum distance (in yards) Marshall can throw the ball? Approximate gravity as 10 yards per second per second.
- $$R(\Theta) = \frac{2v_0^2 \sin(\Theta) \cos(\Theta)}{g}$$
- a) 95 b) 45 c) 38 d) 19 e) NOTA