

# Calculus Individual Test

## FAMAT State Convention 2003

*For all questions, answer E. NOTA, means "none of the above answers is correct".*

1. Given that  $F(x) = \frac{4}{x}$ , choose the correct statement:

- a) The graph of  $F$  is concave up on  $(-\infty, 0)$ .
- b) The graph of  $F$  is concave down on  $(-\infty, 0)$ .
- c) The graph of  $F$  is concave up on  $(-\infty, \infty)$ .
- d) The graph of  $F$  is concave down on  $[0, \infty)$ .
- e) NOTA

2. Use differentials to approximate  $\sqrt{3.3}$ . Also, use the fact that  $\sqrt{4} = 2$ .

- a) 1.825   b) 1.817   c) 1.750   d) 1.650   e) NOTA

3. Find  $f(x)$  if  $f''(x) = x^2$ ,  $f'(0) = 7$  and  $f(0) = 2$ .

- a)  $x^2 + 9$    b)  $\frac{1}{12}x^4 + 7x + 2$    c)  $x^2 + 7x + 2$
- d)  $x^4 + 84x + 24$    e) NOTA

4. Choose the correct statement given that

$$\int_0^9 f(x) dx = 5 \text{ and } \int_3^9 f(x) dx = -1.$$

- a)  $\int_3^0 f(x) dx = 6$    b)  $\int_9^3 f(x) dx = -1$
- c)  $\int_0^3 f(x) dx = 4$    d)  $\int_0^3 f(x) dx = 6$    e) NOTA

5. Evaluate:  $\int x\sqrt{x+1} dx$

- a)  $\frac{3x+2}{2\sqrt{x+1}} + c$    b)  $\frac{2}{15}(x+1)^{3/2}(3x-2) + c$
- c)  $\frac{1}{4}(x+1)(x-1) + c$    d)  $\frac{1}{3}x^2(x+1)^{3/2} + c$
- e) NOTA

6. The value of  $\lim_{x \rightarrow \infty} \frac{3}{x^2}(2 + 4 + 6 + \dots + 2x) =$

- a) 2   b) 3   c) 4   d) 6   e) NOTA

7. When the area in sq. units of an expanding circle is increasing twice as fast as its radius in linear units, the radius is

- a)  $\frac{1}{4\pi}$    b)  $\frac{1}{4}$    c)  $\frac{1}{\pi}$    d) 1   e) NOTA

8. If  $f'(x)$  and  $g'(x)$  exist and  $f'(x) > g'(x)$  for all real  $x$ , then the graphs of  $f(x)$  and  $g(x)$

- a) intersect exactly once   b) do not intersect
- c) intersect no more than once
- d) may intersect more than once   e) NOTA

2003 Calculus Individual Test page 2

9. What are the coordinates of the inflection point on the graph of  $y = (x+1)\text{Arc tan } x$ ?

- a)  $(-1,0)$  b)  $(0,0)$  c)  $(0,1)$  d)  $(1,1)$  e) NOTA

10. The slope of a curve at  $(x, y)$  is

$\frac{dy}{dx} = (x-2)(x-3)$ . At the point where  $x = 2$ , the curve has a:

- a) vertical tangent b) point of inflection  
c) minimum d) maximum e) NOTA

11. A conical tank is being filled with water at a rate of  $12 \text{ ft}^3 / \text{min}$ . The rate of change of the depth of the water is 4 times the rate of change of the radius of the water surface. At the moment when the depth is 8 ft. and the surface radius is 2 ft., the area of the surface is changing at a rate of

- a)  $\frac{3}{4\pi} \text{ sq ft/min}$  b)  $3 \text{ sq ft/min}$  c)  $4 \text{ sq ft/min}$   
d)  $4\pi \text{ sq ft/min}$  e) NOTA

12. If the line  $3x - 4y = 0$  is tangent in the first quadrant to the curve  $y = x^3 + k$ , then  $k$  is

- a)  $\frac{1}{2}$  b)  $\frac{1}{4}$  c)  $0$  d)  $-\frac{1}{2}$  e) NOTA

13. If  $y = \ln(x^2 + y^2)$ , then the value of  $\frac{dy}{dx}$  at the point  $(1, 0)$  is

- a)  $0$  b)  $\frac{1}{2}$  c)  $1$  d)  $2$  e) NOTA

14. Find the area of the region enclosed by  $y^3 = x^2$  and the chord joining  $(-1, 1)$  and  $(8, 4)$ .

- a)  $\frac{45}{2}$  b)  $\frac{27}{10}$  c)  $\frac{64}{3}$  d)  $\frac{121}{12}$  e) NOTA

15. A solid has an elliptical base with major axis 6 units and minor axis 3 units. Each cross section  $\perp$  to the major axis is a square. One side of the square is a chord of the ellipse parallel to the minor axis. Find the volume of this solid (in cubic units).

- a) 24 b) 36 c) 48 d) 64 e) NOTA

16. Which of the following converge?

I.  $\int_0^1 x^{-2} dx$  II.  $\int_0^2 \frac{\ln x}{x} dx$  III.  $\int_1^{\infty} e^{-x} dx$

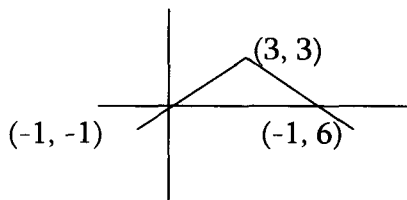
- a) I only b) II only c) III only  
d) I & II only e) NOTA

17. A vertical cylindrical tank has liquid flowing into it at the rate of  $6 \text{ cu.ft./min}$ . How fast is the level of the liquid rising if the radius of the tank is 6 feet?

- a)  $\frac{1}{2\pi}$  b)  $\frac{1}{3\pi}$  c)  $\frac{1}{6\pi}$  d)  $\frac{1}{12\pi}$  e) NOTA

2003 Calculus Individual Test page 3

18. The graph of  $f''$  is shown below. If  $f''(0) = f''(4) = 0$ ,  $f''(2) = 2$ , and  $f'(2) = 3$ , then find  $f'(4)$ .



- a)  $\frac{1}{2}$    b) 4   c) 5   d) 7   e) NOTA

19.  $\int_0^{\infty} x e^{-2x} dx$  converges to

- a)  $-\frac{1}{4}$    b)  $\frac{1}{4}$    c)  $\frac{1}{2}$    d) it diverges   e) NOTA

20. The  $x$  &  $y$  coordinates of a moving particle are described by  $y = x^2$ . At what point on the curve are both coordinates changing at the same rate?

- a)  $(\frac{1}{2}, \frac{1}{4})$    b) (1, 1)   c) (2, 4)  
d) (0, 0)   e) NOTA

21. A differential equation's solution is roughly approximated using Euler's Method and a step of  $\Delta x = -1$ . If  $f'(x) = x^2 - x$  and  $f(1) = 2$ , then  $f(-2) \approx$

- a) -2   b) 0   c) 1   d) 2   e) NOTA

22. Coolant is escaping from a radiator at a rate of  $R(t) = \frac{40}{1+t^2}$   $\text{cm}^3/\text{min}$  where  $t$  is measured in minutes. How many cubic centimeters escape during the first minute?

- a) 10   b)  $10\pi$    c) 12   d)  $12\pi$    e) NOTA

23. The relation between the period  $P$  and length  $L$  of a pendulum is given by  $P(L) = 2\pi\sqrt{\frac{L}{9.81}}$ . If the length of a large pendulum's steel cable is decreasing at the rate of  $0.1$  m/s, what is the rate of change of the period  $P$  of the pendulum at the moment the cable's length  $L$  reaches 3 meters? Round your answer to the nearest thousandth.

- a) -0.022   b) -0.001   c) -0.579  
d) -0.058   e) NOTA

24. If  $h(x) = (x-k)^{1/3}(2x-k)^{2/3}$  for  $k > 0$ , the point  $(\frac{k}{2}, h(\frac{k}{2}))$  defines the location of

- a) a vertical tangent   b) a relative minimum  
c) a cusp   d) a discontinuity   e) NOTA

## 2003 Calculus Individual Test page 4

25. The function  $f(x) = \frac{3}{1+e^{\frac{1}{x}}}$  has

- I. A horizontal asymptote,  $y = 1.4$
- II. A non-removable discontinuity at  $(0, f(0))$
- III. A maximum value of 3

- a) I only      b) II only      c) II & III only  
d) I & II only      e) NOTA

26. Which of the following is a zero of the

function  $F(x) = \int_{-1}^x \sin r dr$ ?

- a) 0      b) 1      c)  $\frac{\pi}{2}$       d)  $\pi$       e) NOTA

27. If  $\begin{cases} x(t) = 2 \sin t \\ y(t) = \cos 2t \end{cases}$  then  $\frac{d^2 y}{dx^2} =$

- a)  $-2 \sin t$       b)  $1 - \tan^2 t$       c)  $-2 \cos t$   
d)  $-1$       e) NOTA

28. If  $\int_{-c}^1 g(x+c) dx = 10$  for a constant  $c$ , then

$$\int_{1+c}^0 g(x) dx = ?$$

- a)  $-10$       b)  $10$       c)  $10 - c$   
d)  $c - 10$       e) NOTA

29. The diagonal of an expanding square is increasing at the rate of  $d'(t) = 2t$  in/sec. If  $s'(t)$  represents the rate of change of a side of the square, then  $s'(t)$  is

- a) linear and  $s'(t) > d'(t)$   
b) linear and  $s'(t) < d'(t)$   
c) nonlinear and  $s'(t) > d'(t)$   
d) nonlinear and  $s'(t) < d'(t)$   
e) NOTA

30. The amount,  $A$ , of a radioactive substance is given by the function  $A(t) = 4e^{\frac{-t}{7}}$ . The average amount of substance present over the time interval  $[0, 4.852]$  is approximately (round to the nearest thousandth).

- a)  $-0.412$       b)  $-2.000$       c)  $2.885$   
d)  $3.981$       e) NOTA