Team Questions—Calculus Bowl 2003
FAMAT State Convention

1. For $F(x)=\int_{-3}^{x} \ln w^{3}\,dw$, find $\frac{d^2F}{dx^2}$.

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2. Given $F(x)=\frac{3x^3-3x^2-6x}{2x^5-2x}$ and the five statements below. If 1 = a TRUE statement and 0 = a FALSE statement, what is the sum determined by the statements below?

i) x = 1 is a an asymptote of F ii) F has a root at x = 2
iii) y = 0 is an asymptote of F

iv) x = 0 is an asymptote of F v) x = -1 is NOT an asymptote of F

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3. If $\int_{m}^{n} \cos x\,dx = 0$, find the ordered pair (m, n), if $-2\pi \le m \le 2\pi$ and $-2\pi \le n \le 2\pi$, and n>m.

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4. Let A = $\lim_{x\to\pi} \frac{f(x)}{g(x)}$ if $f'(x)=3x$, $g'(x)=\sin x$, and $f(0)=g(0)=0$.

Let B = $\lim_{h\to 0} \frac{\int_{1}^{1+h} \sqrt{x^3+8}\,dx}{h}$

Find A + B. Express your answer to the nearest thousandth.

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5. If $f(x)=\sin e^{x}$ for $0\le x\le 1$, and c such that 0 < c < 1, and $f'(c)=f(1)-f(0)$, then find c to the nearest thousandth.

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6. Let A be the volume of the region in quadrant I enclosed by $y=-x^2+3$ when it is rotated about the x-axis.

Let B be the volume of the region in quadrant I enclosed by $y=-x^2+3$ when it is rotated about the y-axis.

Find the positive difference between A and B (Express your answer to the nearest thousandth).

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7. If $y' = y\cos x$ is solved numerically using Euler's Method and steps of $\Delta x = \frac{\pi}{2}$, and the initial condition is y(0) = 1, then find the exact value of the error in the approximation of $y(\pi)$.

8. A pond in a zoo is studied as a frog population, P(t), is introduced, where t is in months. Over a period of 3 years, the population is counted and recorded on a monthly basis. The population is found to increase according to the equation, $P(t) = 50 + \frac{450}{1 + 950e^{-.35t}}$. After how many months does the rate of change in the number of frogs become the greatest? Express your answer to the nearest thousandth.

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9. A cup of hot 170-degree soup is found to cool at the rate of $T'(t) = -0.3(T - 72)$ degrees/min, where t is the current temperature of the soup. What is the temperature of the soup after 4 minutes (round to the nearest thousandth)?

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10. Let $y^2 - 14xy + 112x^2 = 16$ define a curve.

Find the sum of all the slopes of all the tangent lines to the curve at x = ½ .

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11. The area enclosed by the intersection of $f(x) = -x^2$ and $g(x) = 2x - 2$ is to be divided into 3 sections of equal area using two vertical lines, $x = A$ and $x = B$.

Find A + B.

12.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	-2	1	4	-1
1	2	4	-3	5
2	-3	-1	1	2
3	3	1	-1	4

I. If $k(x) = g(f(x))$, then $k'(1) = A$. II. If $k(x) = g(x^2)$, then $k'(1) = B$. III. If $k = f^{-1}$, then $k'(2) = C$.

Find A + B + C.

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13. A 17-year-old calculus student decides to set aside $20/month in a fund that pays an annual interest rate of 8% (compounded monthly). If he

maintains this savings plan for 40 years, how much money, to the nearest dollar, will be in the fund?

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14. At time t = 0, a particle is temporarily moving with a velocity of 8 m/min and its position is x (0) = 0. One minute later, its velocity has dropped to 6 m/min. Its acceleration is inversely proportional to its velocity. Determine the total distance traveled by the particle on its domain. Express your answer to the nearest thousandth.