BC Calculus Topic Test
FAMAT State Convention 2003

For all questions, "E. NOTA" means none of the above answers is correct. Unless otherwise stated, assume all numbers are real.

1. Evaluate: $\int_0^{\pi} \cos^2 \lambda \, d\lambda$

A. 0
B. $\frac{\pi}{2}$
C. π
D. 2π
E. NOTA

2. Let a curve be given parametrically by $x = t^2 \cos t + 2t$ and $y = t^3 - (t+1)\ln(t+1) + 2$. Find $\frac{dy}{dx}$ at the point $(0,2)$ in the *xy*-plane.

A. –1
B. $-\frac{1}{2}$
C. 0
D. 2
E. NOTA

3. $\lim_{x \to 0} \frac{\sin x \cdot \cos x}{x}$

A. -1
B. 0
C. 1
D. Does Not Exist
E. NOTA

4. Use Euler's method to approximate $y(1.2)$ for the differential equation $2xy' + y = 0$ with initial condition $y(1) = 1$ and step-size $\Delta t = .1$. Round your answer to the nearest thousandth.

A. .869
B. .907
C. .913
D. .950
E. NOTA

5. Which of the following converge?

I. $\sum_{n=0}^{\infty} \frac{n^2}{n^2+1}$

II. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n}$

III. $\sum_{n=0}^{\infty} \frac{3^n}{n!}$

A. III only
B. I and II
C. I and III
D. I, II, and III
E. NOTA

6. Let a particular solution to the differential equation $(2y-6)dy - (8x+8)dx = 0$ be the intersection of two lines, y_1 and y_2. Find $y_1(1) + y_2(1)$.

A. –2
B. 0
C. 3
D. 6
E. NOTA

7. Which of the following best describes the convergence of $\sum_{n=1}^{\infty} \frac{n^2 - n + n^6 - e^4 n^3}{\pi n^3 - 6n^4 + n^7}$.

A. Absolutely convergent
B. Conditionally convergent
C. Uniformly convergent
D. Divergent
E. NOTA

8. Let a curve *C* be given parametrically by $x = 3\cos t$ and $y = 3\sin t$. Find the length of *C* from $t = 0$ to $t = 3$.

A. 2
B. 4
C. 6
D. 8
E. NOTA

9. Use a Maclaurin Series to evaluate $\int \frac{\sin x}{x} dx$

A. $1-\frac{x^2}{3!}+\frac{x^4}{5!}-\frac{x^6}{7!}+...+C$
B. $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+...+C$
C. $1-\frac{x^2}{3\cdot 3!}+\frac{x^4}{5\cdot 5!}-\frac{x^6}{7\cdot 7!}+...+C$
D. $x-\frac{x^3}{3\cdot 3!}+\frac{x^5}{5\cdot 5!}-\frac{x^7}{7\cdot 7!}+...+C$
E. NOTA

10. The definite integral $\int_0^{\frac{\sqrt{3}}{2}} \sqrt{4x^2+1}\, dx$ is transformed to $\int_a^b \frac{\sec^3\theta}{2} d\theta$ using the trigonometric substitution $\tan\theta = 2x$. If $0 \le a < b < \pi$, find $a+b$.

A. $\frac{\pi}{6}$
B. $\frac{\pi}{4}$
C. $\frac{\pi}{3}$
D. 1
E. NOTA

11. Evaluate: $\int x\cos x\, dx$

A. $x\sin x+\cos x+C$
B. $\frac{x^2\sin x}{2}+C$
C. $x\cos x+\sin x+C$
D. $x\sin x-\cos x+C$
E. NOTA

12. Evaluate: $\lim_{x\to\infty}\frac{1}{x}\int_0^x\left(\sqrt{t^2+5t}-t\right)dt$

A. 0
B. 1
C. 2.5
D. 5
E. NOTA

13. Use differentials to approximate the arc length of the graph $y=\cos x$ from $x=\frac{\pi}{4}$ to $x=\frac{\pi}{2}$ to the nearest hundredth.

A. .84
B. .87
C. .96
D. 1.06
E. NOTA

14. Evaluate $\frac{d}{dx}\left(\arccos\frac{1}{x}\right)$ when $x=2$

A. $\frac{1}{5}$
B. $\frac{\sqrt{3}}{6}$
C. $\frac{1}{2}$
D. 1
E. NOTA

15. The shape of a rug is given by the cardioid $r=12+12\cos\theta$, where r is measured in inches. What is the area of the rug in square inches?

A. 108π
B. 216π
C. 324π
D. 432π
E. NOTA

16. Use Maclaurin Series expansions for $\cos x$, e^x, and e^{-x} to determine $\cos i$, where $i=\sqrt{-1}$.
Note: These expansions converge for all complex x.

A. $\frac{e^2+1}{2e}$
B. $\cos e$
C. e^2-1
D. $\frac{\cosh 1}{2}$
E. NOTA

17. Which of the following statements are true concerning $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$?

I. The series is conditionally convergent
II. The terms of the series can be rearranged in a manner such that the series converges and has sum π
III. If the terms are rearranged, the resulting series will always converge
IV. $\sum_{n=1}^{\infty} \left| \frac{(-1)^{n+1}}{n} \right|$ converges absolutely.

A. I only
B. I and II
C. I, II, and III
D. I, II, III, and IV
E. NOTA

18. Find the slope of the line tangent to the polar graph $r = \cos\theta$ when $\theta = \frac{\pi}{6}$.

A. $-\frac{\sqrt{3}}{3}$
B. $-\frac{1}{2}$
C. $\frac{1}{2}$
D. $\frac{\sqrt{3}}{2}$
E. NOTA

19. Which of the following is not a solution to the differential equation $dy = \frac{\sin\theta}{\cos^3\theta} d\theta$

A. $y = 2\tan\theta\sec\theta + C$
B. $y = \frac{1}{2\cos^2\theta} + C$
C. $y = \frac{\sec^2\theta}{2} + C$
D. $y = \frac{\tan^2\theta}{2} + C$
E. NOTA

20. Evaluate: $\lim_{x\to 0^+} x^3 \ln x$

A. $-\frac{1}{3}$
B. 0
C. 1
D. $+\infty$
E. NOTA

21. $\int_0^{\frac{\pi}{2}} 5\sin^3 x\cos^3 x\,dx = \frac{m}{n}$, where m and n are relatively prime. Find $m+n$.

A. 5
B. 9
C. 13
D. 17
E. NOTA

22. The graph of $3x^3 - 4x^2y + 6y^2 - 3x - 6y + 4 = 0$ is rotated clockwise $45°$. Find $\frac{dy}{dx}$ of the new graph at the point $\left(\sqrt{2}, 0\right)$.

A. $-\frac{7}{23}$
B. 0
C. 1
D. $\frac{15}{8}$
E. NOTA

23. Find the sum of all a on $[0, 2\pi]$ such that $\int_0^a \left(\cos^4 x - \sin^4 x\right) dx = \frac{1}{4}$.

A. $\frac{\pi}{2}$
B. π
C. 2π
D. 3π
E. NOTA

24. Find the interval of convergence for $\sum_{n=2}^{\infty} \frac{x^n}{\ln n}$.

A. $(-1,1)$
B. $(-1,1]$
C. $[-1,1)$
D. $[-1,1]$
E. NOTA

25. Evaluate: $\int_0^2 \frac{1}{(x-1)^3}\,dx$

A. 0
B. $\frac{1}{2}$
C. 2
D. The integral diverges
E. NOTA

26. Let the position of a particle in a plane be given by the vector-valued function $\mathbf{r}(t) = \sin t\,\mathbf{i} + \left(6t^5 + 4t^3 + 4t - 8\right)\mathbf{j}$ for all $t \in \mathbb{R}$. What is the speed of the particle when $t = 0$?

A. 0
B. 1
C. 2
D. $\sqrt{17}$
E. NOTA

27. Evaluate: $\int x^5\sqrt{x^2+16}\,dx$

A. $\frac{x^6\left(x^2+16\right)^{\frac{3}{2}}}{9} + C$

B. $\frac{x^5\left(x^2+16\right)^{\frac{3}{2}}}{15} + C$

C. $\frac{\left(x^2+16\right)^{\frac{7}{2}}}{7} - \frac{32\left(x^2+16\right)^{\frac{5}{2}}}{5} + \frac{216\left(x^2+16\right)^{\frac{3}{2}}}{3} + C$

D. $\frac{\left(x^2+16\right)^{\frac{3}{2}}\left(15x^4 - 192x^2 + 2048\right)}{105} + C$

E. NOTA

28. $\int_0^1 \frac{8\text{Cos}^{-1}x}{\sqrt{1-x^2}}\,dx = a$. Find $\log_\pi a$.

A. 0
B. 1
C. 2
D. 3
E. NOTA

29. Which of the following is true concerning the logistic equation $\frac{dP}{dt} = kP\left(1 - \frac{P}{L}\right)$ and the population $P(t)$ of an environment at time t?

I. L is the carrying capacity of the environment
II. The graph of $P(t)$ can be approximated by an exponential function for large values of P
III. If $P(t) > L$, the population will decrease
IV. The population has its largest rate of increase when $P(t) = \frac{L}{2}$

A. I and III
B. I, II, and III
C. I, III, and IV
D. I, II, III, and IV
E. NOTA

30. Find the value of $\int_{a+1}^{\infty} \frac{1}{x^2 - a^2}\,dx$ if a is a positive constant.

A. $\frac{\ln(2a-1)}{2a-2}$
B. $\frac{\ln(2a+3)}{2a+2}$
C. $\frac{\ln(2a+1)}{2a}$
D. The integral diverges
E. NOTA