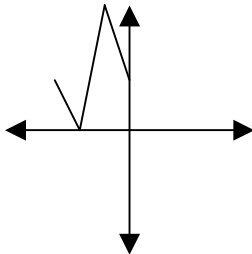


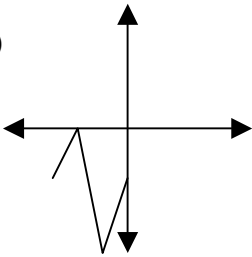
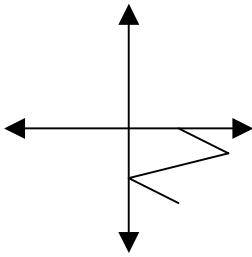
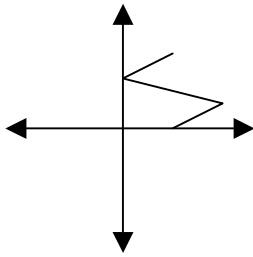
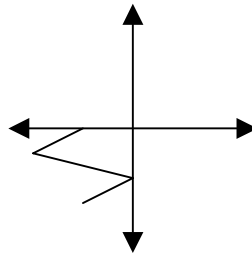
ALPHA ANALYTIC GEOMETRY  
FAMAT State convention 2003

For all questions, e) NOTA means none of the above answers is correct

1 List all the asymptotes of the function:  $f(x) = \frac{2}{x - \log_x 4}$

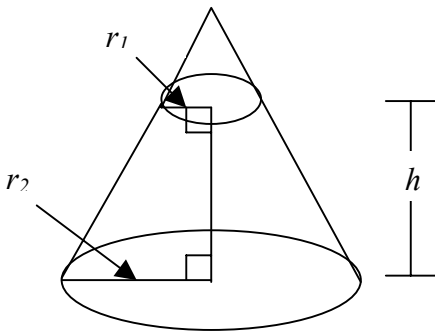
- a)  $x = 0, x = 2, y = 0$       b)  $x = 2, y = 0$       c)  $x = 2, x = 0$       d)  $x = 0, y = 0$       e) NOTA

2 if  $f(x)$  is represented by:  then which of the following most closely represents  $-f^{-1}(x)$ ?

- a)       b)       c)       d) 

e) NOTA

3 Find the volume of the frustum of the cone where  $r_1 = 2, r_2 = 6$ , and  $h = 3$ .



- a)  $54\pi$   
b)  $\frac{104\pi}{3}$   
c)  $36\pi$   
d)  $156\pi$   
e) NOTA

4 If  $x = 3\sin(t)$ ,  $y = 5\cos(t)$ , and  $r^2 = x^2 + y^2$ , then what is the minimum value of  $r$  over  $t \in [0, 2\pi]$ ?

- a) 4      b) 3      c) 16      d) 9      e) NOTA

5 I am holding a convex pentagon-shaped piece of cardboard. If I placed this piece of cardboard over a coordinate plane, its vertices would be at  $(0,0)$ ,  $(2,0)$ ,  $(3,2)$ ,  $(2,4)$ , and  $(0,3)$ . At each vertex is placed a 10 lb weight, except at the vertex represented by the coordinate  $(0,3)$  where there is suspended a 30 lb weight. If I wished to balance the weighted cardboard on a pencil, at which point should I place the pencil?

- a)  $\left(2, \frac{9}{5}\right)$     b)  $\left(\frac{9}{5}, 2\right)$     c)  $\left(1, \frac{15}{7}\right)$     d)  $\left(\frac{9}{7}, \frac{17}{7}\right)$     e) NOTA

6 If  $f(x) = e^x \cos(x)$  and  $g(x) = e^x \sin(x)$ , find the zeros of the function  $h(x) = \frac{f^2 - g^2}{f^2 + g^2}$  in the domain  $[0, 2\pi]$ .

- a)  $\frac{\pi}{4}, \frac{5\pi}{4}$     c)  $\frac{\pi}{2}, \frac{3\pi}{2}$     d)  $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$     e) No Solutions In This Domain    e) NOTA

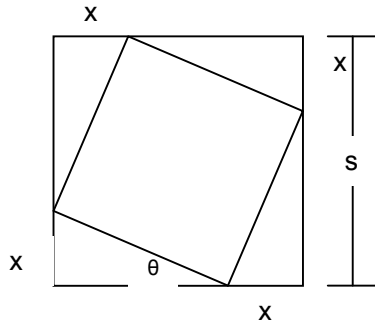
7 If  $P_n$  represents the  $n^{\text{th}}$  Pyramidal number and  $P_1 = 1$ , then find  $P_{100} - P_{99}$ .

- a) 100    b) 10100    c) 4050    d) 5050    e) NOTA

8 How many 'petals' does the graph of the curve  $r = 4 \cos \theta \sin \theta$  have?

- a) 2    b) 3    c) 4    d) 6    e) NOTA

9 You are given a square with sides of length  $s$ . Four points, one at each vertex, start moving clockwise along the sides of the square (at the same rate). In the picture below, the points have already moved a distance of  $x$  away from the vertices, forming the angle  $\theta$ . Give an equation, in terms of  $x$  and  $\theta$ , for the area of this new square formed by the four points.



- a)  $A = (x \csc \theta)^2$     b)  $A = \frac{x^2}{\cos^2 \theta}$     c)  $A = \frac{x}{\sin \theta}$     d)  $A = x \tan^2 \theta$     e) NOTA

10 What is the parametric representation of a line through the points  $(1,3)$  and  $(5,6)$ ?

- a)  $x = t$   
 $y = 3t^2 - 69t + 69$     b)  $x = t + 1$   
 $y = 3\left(\frac{t}{4} + 1\right)$     c)  $x = t + 1$   
 $y = t$     d)  $x = 4t - 3$   
 $y = t^2 + 2$     e) NOTA

11 Link takes a closer look at his recently acquired equilateral Triforce (Fig.1) and notices that it is slightly more intricate than he had thought (Fig. 2). This pattern, he notices, is continuing on “infinitely” (Fig. 3, etc). If the sides of the Triforce (in each figure) is 1 unit in length, then what is the limit of the shaded area of the Triforce if this pattern continues infinitely?



Fig

1. Fig 2

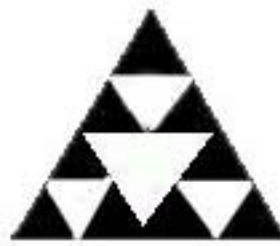
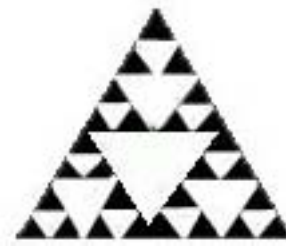


Fig. 3



- a)  $\frac{3\sqrt{3}}{16}$       b)  $\frac{\sqrt{3}}{4}$       c) 0      d) 1      e) NOTA

12 The Florida Gators football team in 2002 has experienced exponential decay. If they started at 100% performance on August 31 and were performing at 50% on October 12 (42 days later), how many more days (rounded to the nearest 10<sup>th</sup> of a day) will it take the Gators to reach 5%?

- a) 181.5      b) 79.8      c) 139.5      d) 42      e) NOTA

13 The University of Miami Hurricanes football players' salaries vary quadratically with the number of touchdowns they score. The coordinate (2,2) represents 2 touchdowns earning a player 2 thousand dollars. If a player, during a 3 week period, has crossed the points, (2,2), (3,8), and (4,18), how much will he make during the next week if he scores 5 touchdowns?

- a) 25      b) 28      c) 32      d) 36      e) NOTA

14 What is the distance between the foci of the hyperbola given by the equation  $\frac{x^2}{9} - \frac{y^2}{16} = 1$ ?

- a) 10      b) 6      c)  $2\sqrt{5}$       d)  $2\sqrt{7}$       e) NOTA

15 What is the area of the rectangle formed by the four endpoints of the latera recta of the ellipse given by the equation  $\frac{x^2}{9} + \frac{y^2}{16} = 1$ ?

- a)  $\frac{9\sqrt{7}}{2}$       b)  $9\sqrt{7}$       c) 45      d)  $\frac{9\sqrt{5}}{2}$       e) NOTA

16 Find the center of the hyperbola given by the equation  $y = \frac{4x-3}{x-1}$ .

- a) (4,1)      b) (1,4)      c) (1,2)      d) (1,3)      e) NOTA

17 Which of the following is an equation of a line tangent to the circle  $x^2 + y^2 = 8$  and containing the point  $(6, -2)$ .

- a)  $y = \frac{1}{2}x - 5$       b)  $y = 4 - x$       c)  $y = 2 - \frac{2}{3}x$       d) Both a) and b)      e) NOTA

18 The conic section given by the equation  $2x^2 - xy - 3y^2 - 3x + 2y + 1 = 0$  is most accurately described as:

- a) hyperbola      b) ellipse      c) two concurrent lines      d) two intersecting lines  
e) NOTA

19 Which of the following is the equation of a parabola with focus  $(3, 2)$  and directrix  $x = -1$ ?

- a)  $8x = y^2 - 4y - 4$       b)  $y = \frac{x^2 - 4x + 12}{8}$       c)  $x = 8y^2 - 32y + 33$       d)  $8x = y^2 - 4y + 12$   
e) NOTA

20. A circle has a chord of length 12 that is tangent to a smaller, concentric circle. Find the area between the two circles.

- a)  $6\pi$       b)  $9\pi$       c)  $24\pi$       d)  $36\pi$       e) NOTA

21 Hyperbolic sine and hyperbolic cosine are represented as follows:  $\sinh(x) = \frac{e^x - e^{-x}}{2}$ ,  $\cosh(x) = \frac{e^x + e^{-x}}{2}$ .

What is (are) the asymptote(s) of hyperbolic tangent if  $\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$ ?

- a)  $x = 1$       b)  $y = -1$       c)  $x = \pm 1$       d)  $y = 1$       e) NOTA

22 Which of the following are 4<sup>th</sup> roots of the complex number  $16e^{\frac{4\pi i}{3}}$ .

- i)  $-1 - \sqrt{3}i$       ii)  $1 - \sqrt{3}i$       iii)  $\sqrt{3} - i$       iv)  $1 - \sqrt{3}i$       v)  $-1 + \sqrt{3}i$       vi)  $-\sqrt{3} + i$   
a) i,ii,v,vi      b) i,iii,vi      c) ii,iv,v      d) ii,v, vi      e) NOTA

23 What is the value of  $(1 + i)^{20}$ ?

- a) 256      b) 1024      c)  $-1024 - 1024i$       d)  $-1024$       e) NOTA

24 What is the value of the cosine of the angle between the vectors  $\langle 2, 4, 4 \rangle$  and  $\langle -1, 4, 0 \rangle$ ?

- a)  $\frac{7}{6\sqrt{17}}$       b)  $\frac{9}{7}$       c) 1      d) 38.9      e) NOTA

25 Which of the following is a vector perpendicular to  $\langle 2,3,4 \rangle$  and  $\langle -1,4,0 \rangle$ ?

- a)  $\langle -16,-4,11 \rangle$       b)  $\langle -16,4,5 \rangle$       c)  $\langle 16,4,-11 \rangle$       d) Both A and C      e) NOTA

26 If  $f(x) = \frac{x^2}{9}$  and  $g(y) = \frac{y^2}{16}$ , then what is the sum of the eccentricities of the two conic sections given by the equations:  $f(x) \pm g(y) = 1$ ?

- a)  $\frac{5+\sqrt{7}}{4}$       b)  $\frac{5+\sqrt{7}}{3}$       c)  $\frac{3\sqrt{7}+20}{12}$       d) 1      e) NOTA

27 What are the equations of the directrices of the hyperbola given by the equation  $\frac{x^2}{9} - \frac{y^2}{16} = 1$ ?

- a)  $x = \pm \frac{9}{5}$       b)  $x = \pm \frac{16}{5}$       c)  $y = \pm \frac{9}{5}$       d)  $y = \pm \frac{16}{5}$       e) NOTA

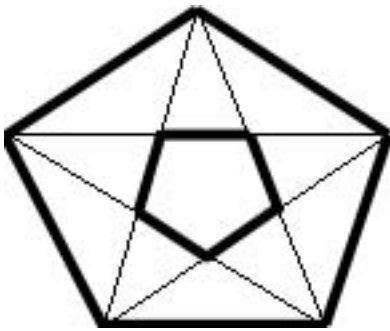
28 A triangle with vertices at the points  $(0,0)$ ,  $(x,y)$ , and  $(-x,2y)$  has an area  $A$ . What is the relationship between  $x$ ,  $y$ , and  $A$ ?

- a)  $A = 3xy$       b)  $3A = 2xy$       c)  $2A = 3xy$       d)  $3A = xy$       e) NOTA

29 What is the distance between the points, given in spherical coordinates,  $(5,30^\circ,60^\circ)$  and  $(2,0^\circ,90^\circ)$ ?

- a)  $\sqrt{15}$       b)  $\sqrt{14}$       c) 4      d)  $\sqrt{17}$       e) NOTA

30 A regular pentagon with sides of length  $s$  has all of its diagonals drawn. A new pentagon, formed by the intersection of the diagonals, has sides of length  $\frac{\sqrt{5}-1}{\sqrt{5}+1}s$ . If the area of the large pentagon is  $A_L$  and the area of the small pentagon is  $A_S$ , then what is the value of  $\frac{A_L}{A_S}$ .



- a)  $\frac{3+\sqrt{5}}{3-\sqrt{5}}$   
 b)  $\frac{6+\sqrt{5}}{6-\sqrt{5}}$   
 c) 5  
 d)  $\frac{1}{5}$   
 e) NOTA

