Logs & Exponents - Alpha

FAMAT State Convention - 2003

For all questions (except #8), answer E. "NOTA" means none of the above answers are correct.

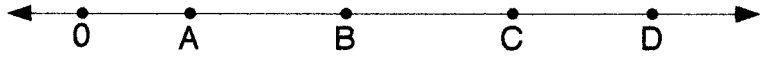
1. Find a decimal approximation correct to four decimal places for $\log_{12} 35$.

   A. 0.6989 B. 1.4308 C. 1.5441 D. 3.5553 E. NOTA

2. Given: $\log_b a = 13.5$, where b is an acceptable base for a logarithm. To four decimal places, what would be the logarithm base b of the fifth root of a?

   A. 1.1303 B. 1.6829 C. 2.6027 D. 2.7000 E. NOTA

3. On the given number line, the coordinate of point A is $\log_b x$, the coordinate of point B is $\log_b y$, the coordinate of point C is $\log_b x^3$, & the coordinate of point D is $\log_b y^3$ where b is an acceptable constant for the base of a logarithm and x is greater than 1. In relation to the lengths of $\overline{CD}$ and $\overline{AB}$, which of the following must be true?



Note: Figure not to scale

   A. the length of $\overline{AB}$ = 3 times the length of $\overline{CD}$
   B. 3 times the length of $\overline{AB}$ = the length of $\overline{CD}$
   C. the length of $\overline{AB}$ = the length of $\overline{CD}$
   D. It is not possible to determine relative size from the information given. E. NOTA

4. Given: $x \in \left(0, \frac{\pi}{2}\right)$. Solve $\ln(\sin x) - \ln(\cos x) = 1$.

   A. Arctan e B. Arctan π C. e D. π E. NOTA

5. 43^{-1} turned into a decimal is .02325581.... This decimal repeats with a regular period. What is the last digit of the period (before the decimal starts to repeat)?

   A. 1 B. 2 C. 3 D. 9 E. NOTA

6. Given: $\begin{cases} \log_x 9 + \log_8 y = \frac{7}{3} \\ \log_9 x + \log_y 8 = \frac{7}{2} \end{cases}$ One of the solutions for this system of equations is...

   A. $\left(\frac{1}{2}, 3\right)$ B. $\left(2, \frac{1}{3}\right)$ C. (8,9) D. (729,64) E. NOTA

7. If $\log_b t = 8.64$ give an exact numerical value for $\log_b \sqrt[4]{t^3 b^2}$.

A. 0.93651374248 B. 2.1564025828 C. 6.48 D. 6.98 E. NOTA

8. $2^{10000} + 2^{1000}$ is closest to which of the following on the real number line?

A. 2^{10000} B. 2^{11000} C. 2^{10001} D. 4^{10001}

9. Simplify : 2003^{-2003}. Give your answer exactly or in scientific notation to four significant digits.

A. 0 B. 5.408×10^{-6614} C. 4.734×10^{-6599} D. 2.003×10^{-2006} E. NOTA

10. If 2003^{2003} (base 10) were changed to a number in base 7, how many digits would it have?

A. 3 B. 1849 C. 6613 D. 7826 E. NOTA

11. An exponential function's equation is $f(x) = a \times b^x$. If $0 < b < 1$, then the graph of $f(x)$ has...

A. no asymptotes B. a vertical asymptote
C. a slant(oblique) asymptote with a positive slope
D. a slant(oblique) asymptote with a negative slope E. NOTA

12. When a ham is placed in an oven preheated to 350°F, the difference between the temperature of the center of the ham and the temperature of the oven is an exponential function of time. If the temperature of the center of the ham is 45°F when it is placed in the oven and has raised to 71°F after 15 minutes, To the nearest minute, how long will it take for the ham to cook if the ham is done when the internal temperature reaches 185°F?

A. 46 min B. 47 min C. 1 hr 43 min D. 2 hr 51 min E. NOTA

13. If $\log_b 125 = c$, then $\log_b 25$ is what percent of c?

A. 20 B. $33\frac{1}{3}$ C. 50 D. $66\frac{2}{3}$ E. NOTA

14. The two solutions to the equation, $x^2 \log 2 + x = \log 4096 + x \log 5$, are a & b. $a + b = ?$

A. 1 B. -1 C. 2 D. 3 E. NOTA

15. How many zeros are there between the decimal and the first non-zero digit in 4000^{-4000}?

A. 14406 B. 14407 C. 14408 D. ∞ E. NOTA

16. Given: $f(x) = \log|x|$ and $g(x) = 2\sin x$. How many points of intersection are there between the two graphs?

A. 30 B. 32 C. 64 D. 65 E. NOTA

17. If b is an acceptable base for a log, $\frac{1}{\log_{b^2} 10}$ is equivalent to all of the following except...

A. $-2\log\left(\frac{1}{b}\right)$ B. $\log b^2$ C. $\log_b \sqrt{2} + \log_b \sqrt{5}$ D. $2\log b$ E. NOTA

18. Solve for x. $\log_6(3x-1) = \log_6\left(\frac{1}{x+2}\right) + 1$

A. $x \in \{1\}$ B. $x \in \{-2\}$ C. $x \in \{3\}$ D. $x \in \{\frac{8}{3}, 1\}$ E. NOTA

19. If x and y are positive reals and $x^y + x^y = x^{y+1}$, then what must be the value of x?

A. 1 B. 2 C. 3 D. impossible to determine E. NOTA

20. Given $\log_b 2 = d$, $\log_b 3 = e$, $\log_b 5 = f$, $\log_b 7 = g$, $\log_b 11 = h$, $\log_b 13 = i$, $\log_b 17 = j$, $\log_b 19 = k$. In terms of d, e, f, g, h, i, j, & k what is $\log_b(22!)$?

A. $d^{19}e^{17}f^{13}g^{11}h^7i^5j^3k^2$ B. $d^{19}e^{10}f^4g^3h^2ijk$ C. $d^{19}e^9f^4g^3h^2ijk$ D. $d^{15}e^9f^4g^3h^2ijk$ E. NOTA

21. To the nearest hundredth of a year, how long would it take for \$8932.21 to double if it were left in an account that pays 5.25% annual interest compounded monthly?

A. 13.54 B. 13.55 C. 13.29 D. 13.23 E. NOTA

22. If 2003! were written in base 12, how many zeros would there be if you started at the right of the number and counted zeros to the left until coming to the first non-zero digit?

A. 997 B. 1000 C. 1999 D. 3998 E. NOTA

23. Fill in the blank. To convert log 17 to ln 17, log 17 must be _?_ ln 10.

A. multiplied by B. subtracted from C. added to D. divided by E. NOTA

24. Given: $e^k = \left(\frac{1}{2}\right)^{\frac{x}{2}}$. What is x in terms of k?

A. $\frac{1}{2}\ln\frac{k}{2}$ B. $-2k\log_2 e$ C. $\frac{2k}{\ln 2}$ D. $2e^k$ E. NOTA

25. $\frac{1}{\log_4 18} + \frac{1}{2\log_6 3 + \log_6 2} + \frac{5}{\log_3 18} =$

A. 1 B. 2 C. 3 D. 4 E. NOTA

26. To the nearest hundredth, what is the solution to the equation: $x^{x^{x^{\cdots}}} = 2$?

A. 1.21 B. 1.41 C. 2.78 D. 4 E. NOTA

27. Given: $f_1(x) = e^x; f_2(x) = ax + b, a > 0 \& b > 0; f_3(x) = ax^2, a > 0; f_4(x) = \log_b x, b > 1$. If x a real number such that x is greater than any solution to $f_i(x) = f_j(x)$ for $i \in \{1,2,3,4\}$ and $j \in \{1,2,3,4\}$ & $i \neq j$, then

A. $f_1(x) > f_3(x) > f_4(x) > f_2(x)$ B. $f_1(x) > f_3(x) > f_2(x) > f_4(x)$
C. $f_3(x) > f_1(x) > f_4(x) > f_2(x)$ D. relationship cannot be determined E. NOTA

28. The Richter scale is a measurement of the intensity of an earthquake. If the Richter scale simply reports the base 10 logarithm of the intensity, then to the nearest tenth, how many times greater was the earthquake in San Francisco that measured 8.5 on the Richter scale than the one in San Diego which measured 7 on the Richter scale?

A. 1.2 B. 8.5 C. 17.1 D. 31.6 E. NOTA

29. Given: $82.3 = 51.4 + 6.1 \times 2.3^{3.4x}$. To the nearest tenth, $x = ?$

A. 0.6 B. 1.9 C. 5.1 D. 30.9 E. NOTA

30. Simplify: $81^{\log_9 19}$.

A. 9 B. 19 C. 81 D. 361 E. NOTA