

**Alpha Individual Test
FAMAT State Convention 2003**

**For all questions, answer E. "NOTA" means
none of the above answers is correct**

1. Find the sum of all the solutions to the equation $2\tan x = 4\sin x$ so that $0 < x < 2\pi$

- a. π b. 2π c. 3π d. 4π

2. If the asymptotes of $y = \frac{4-x^2}{3+x}$, are expressed in the following forms $x=a$ and $y=mx + b$, assuming both exist, find $a+b+m$.

- a. -1 b. 1 c. 5 d. 6

3. Find the range of the following function: $y = \frac{5^x + 5^{-x}}{4}$.

- a. $y \geq 0$ b. $y \geq -0.25$ c. $y \geq 0.25$ d. $y \geq 0.5$

4. Simplify: $\frac{[3(x+2)]!}{(3x+4)!}$

- a. $3x+6$ b. $3x+5$ c. $9x^2+39x+42$ d. $9x^2+33x+30$

5. Find the number of points of intersection of the graphs $y = 2^x$ and $y = \log_6(x-3)$.

- a. 0 b. 1 c. 2 d. 3

2003 Alpha Individual Test page 3

11. If function f is defined for all real numbers x and $f(2) = 5$ and $f(x) = f(x-1) + x + 1$, find $f(10)$.

- a. 44 b. 54 c. 65 d. 77

12. In Ed's Pre Calc class, 60% of the students have brown hair, 30% have brown eyes, and 10% have both brown eyes and brown hair. If a student has brown eyes, what is the probability that that student does not have brown hair?

- a. 0.1 b. 0.2 c. 0.3 d. 0.75

13. Given $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = x + 4y$ and $\tan^{-1}(1) = 2x - y$, find $x + y$ in radians.

- a. $\frac{7\pi}{12}$ b. $\frac{1}{6}\pi$ c. $\frac{1}{7}\pi$ d. $\frac{7\pi}{36}$

14. One of the roots of $x^4 - 10x^3 + 29x^2 - 10x - 50 = 0$ is $3+i$. What is the sum of all the remaining real roots?

- a. 4 b. -4 c. 10 d. -10

15. Identify the vector perpendicular to vectors a and b given $a = (5,2,3)$ and $b = (-2,5,0)$.

- a. (15,-6,21) b. (-6,21,15) c. (-6,29,-15) d. (15,6,-29)

16. Find the sum of all real values for x given $e^x - 20e^{-x} = 8$.

- a. $\ln 2$ b. $\ln 10$ c. $\ln 20$ d. no real roots

2003 Alpha Individual Test page 3

11. If function f is defined for all real numbers x and $f(2) = 5$ and $f(x) = f(x-1) + x + 1$, find $f(10)$.

- a. 44 b. 54 c. 65 d. 77

12. In Ed's Pre Calc class, 60% of the students have brown hair, 30% have brown eyes, and 10% have both brown eyes and brown hair. If a student has brown eyes, what is the probability that that student does not have brown hair?

- a. 0.1 b. 0.2 c. 0.3 d. 0.75

13. Given $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = x + 4y$ and $\tan^{-1}(1) = 2x - y$, find $x + y$ in radians.

- a. $\frac{7\pi}{12}$ b. $\frac{1}{6}\pi$ c. $\frac{1}{7}\pi$ d. $\frac{7\pi}{36}$

14. One of the roots of $x^4 - 10x^3 + 29x^2 - 10x - 50 = 0$ is $3+i$. What is the sum of all the remaining real roots?

- a. 4 b. -4 c. 10 d. -10

15. Identify the vector perpendicular to vectors a and b given $a = (5,2,3)$ and $b = (-2,5,0)$.

- a. (15,-6,21) b. (-6,21,15) c. (-6,29,-15) d. (15,6,-29)

16. Find the sum of all real values for x given $e^x - 20e^{-x} = 8$.

- a. $\ln 2$ b. $\ln 10$ c. $\ln 20$ d. no real roots

17. Evaluate: $\lim_{x \rightarrow 3} \frac{x^3 - 27}{3 - x}$

- a. -27 b. 27 c. -9 d. 9

18. Ed places \$2000 in a bank that guarantees a 5.5% interest annually compounded quarterly. Find the minimum number of full years so that Ed's \$2000 will turn into at least \$10000.

- a. 26 b. 28 c. 30 d. 32

19. Find the 100th term of a sequence if the sum of the first n terms is described by $S_n = 3n^2 - 5$.

- a. 597 b. 1197 c. 2394 d. 4788

20. Find the average of the three arithmetic means between x and y

- a. $\frac{x+y}{5}$ b. $\frac{x+y}{4}$ c. $\frac{x+y}{3}$ d. $\frac{x+y}{2}$

21. Find the area of this enclosed region: $2x^2 + 6y^2 - 4x + 24y + 13 = 0$.

- a. $\frac{169}{12} \pi$ b. $\frac{13}{12} \pi$ c. $\frac{13\sqrt{3}}{6} \pi$ d. $\frac{13\sqrt{3}}{12} \pi$

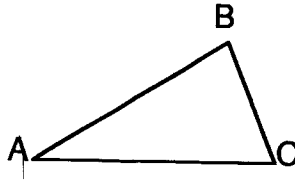
22. Given $y = -3\cos(3x - \frac{\pi}{2})$ and $a =$ amplitude, $b =$ period, $c =$ vertical shift, and $d =$ horizontal shift, find $abcd$.

- a. $\frac{\pi}{3}$ b. $-\frac{\pi}{3}$ c. $\frac{\pi}{6}$ d. $-\frac{\pi}{6}$

23. Find the distance between the polar coordinates $(3\sqrt{2}, \frac{\pi}{4})$ and $(-2\sqrt{2}, -\frac{\pi}{4})$.

- a. $\sqrt{13}$ b. $\sqrt{26}$ c. $5\sqrt{2} + \frac{\pi}{2}$ d. $5\sqrt{2} - \frac{\pi}{2}$

24. Find the smallest possible area of $\triangle ABC$ given $m\angle A = 30^\circ$, $AB = 10$ and $BC = 6$ to the nearest integer.



- a. 30 b. 28 c. 15 d. 13

25. If $x + \frac{1}{x} = 4$ find $x^6 + \frac{1}{x^6}$.

- a. 3 b. 6 c. 2702 d. 3375

26. When \sqrt{i} is expressed as a complex number in polar form it could be equivalent to which of the following choices? Note: $\text{cis } x = \cos x + i \sin x$

- a. $\text{cis } \pi$ b. $\frac{1}{2}\text{cis } \pi$ c. $\text{cis } \frac{\pi}{2}$ d. $\text{cis } \frac{\pi}{4}$

2003 Alpha Individual Test page 6

27. Given $a > 0$ and $b > 0$, find x such that $\log(ab) + \log\left(\frac{1}{ab}\right) = \log_5 x$.

a. 0.25

b. 1

c. 5

d. 25

28. Using binomial expansion, approximate $\sqrt[3]{1.2}$ to the nearest thousandth by adding the first three terms of the expansion.

a. 1.062

b. 1.063

c. 1.064

d. 1.065

29. A target consists of concentric circles with radii of 1, 2, 3, ..., n . What is the area of the n th ring (the last one formed by the region between the two largest circles)?

a. $(n-1)^2\pi$

b. $(2n-1)^2\pi$

c. $(2n-1)\pi$

d. $(2n+1)\pi$

30. Write the equation of the locus of points equidistant from the point $(3,2)$ and the line $x=-3$.

a. $y^2=12(x-2)$

b. $(y-2)^2=12x$

c. $(y-2)^2=6$

d. $x^2=12(y-2)$