

Alpha Topic Test – Equations and Inequalities
FAMAT State Convention 2003

For all questions, answer “E) NOTA” means none of the above answers is correct.

1) Solve for k : $2^{k-5}16^{k+5} = 4^{5k-5}$.

- A) -5 B) $\frac{-1}{5}$ C) $\frac{1}{5}$ D) 5 E) NOTA

2) Find the sum of the reciprocals of the solutions to the equation: $-10x^2 + 41x - 21 = 0$.

- A) 1 B) $\frac{21}{41}$ C) $\frac{41}{10}$ D) $\frac{41}{21}$ E) NOTA

3) X is a real random variable chosen randomly from the interval $[0,20]$ and Y is a real random variable chosen randomly from the interval $[0,10]$. Find the probability (to the nearest hundredth) that $X + Y \leq 7$.

- A) 0.06 B) 0.12 C) 0.25 D) 0.27 E) NOTA

4) Solve for the parameter t : $t - 5 < -t + 2 < 7t + 4$.

- A) $\frac{-1}{4} < t < \frac{7}{2}$ C) $\frac{1}{4} < t < \frac{7}{2}$ E) NOTA
B) $\frac{-7}{2} < t < \frac{1}{4}$ D) $\frac{-7}{2} < t < \frac{-1}{2}$

5) Determine the area of the region enclosed by a circle of radius 1 centered at the origin that also satisfies the inequalities $y < x$ and $y < -x$.

- A) $\frac{\pi}{8}$ B) $\frac{\pi}{6}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{2}$ E) NOTA

6) The angle θ is defined by the following inequalities: $\frac{\pi}{2} \leq \theta < \pi$ and $\cot(\theta) \leq -1$. Solve for θ .

- A) $\frac{3\pi}{4} \leq \theta < \pi$ C) $\frac{3\pi}{4} \leq \theta < \frac{7\pi}{8}$ E) NOTA
B) $\frac{\pi}{2} \leq \theta < \pi$ D) $\frac{7\pi}{8} \leq \theta < \pi$

7) Given that $g(x) = 7x + 3$, what is $g^{-1}(x)$?

- A) $\frac{x-7}{3}$ B) $\frac{x-3}{7}$ C) $\frac{7}{x-3}$ D) $\frac{3}{x-7}$ E) NOTA

8) How many non-negative integers are solutions to: $18x + 5 < 196$.

- A) 9 B) 10 C) 11 D) 12 E) NOTA

9) If l, m, n are solutions to $x^3 - 6x^2 - 19x + 84 = 0$ (where $l < m < n$) find $l^2 + mn^2 + m^2l$.

- A) 1 B) 95 C) 127 D) 159 E) NOTA

10) Find $f(5)$ if $f(1/x) - x^2 = (1+x)f(x)$.

- A) $-\frac{751}{155}$ B) $-\frac{155}{751}$ C) $\frac{155}{751}$ D) $\frac{751}{155}$ E) NOTA

11) Given that $g(x) = \frac{x}{x+1}$, then solve for x in the following: $g(g(g(g(g(g(g(g(g(x)))))))))) = 5$.

- A) $-\frac{10}{44}$ B) $-\frac{5}{44}$ C) $\frac{5}{44}$ D) $\frac{10}{44}$ E) NOTA

12) Find $x + y + z$ if $x\sqrt{y} = 36\sqrt{14}$, $y\sqrt{z} = 336\sqrt{2}$, $z\sqrt{x} = 216\sqrt{2}$.

- A) 138 B) 146 C) 148 D) 154 E) NOTA

13) Given that $\log_5(3x) + \log_{15}(2x^2) = 3.584756$, evaluate $x^2 + 5$.

- A) 30 B) 41 C) 44 D) 54 E) NOTA

14) Solve for x : $4 < \frac{x-5}{5x+7}$.

- A) $\frac{-33}{19} < x < \frac{7}{5}$ C) $\frac{-33}{19} < x < \frac{-7}{5}$ E) NOTA
 B) $\frac{-7}{5} < x < \frac{33}{19}$ D) $\frac{7}{5} < x < \frac{33}{19}$

15) Given that $\frac{3(5x-1)}{x^2-2x-35} = \frac{A}{2x+10} + \frac{B}{2x-14}$, find $A - B$ (A, B are real numbers)

- A) -7 B) -4 C) 4 D) 7 E) NOTA

16) For what value(s) of θ , $0 \leq \theta < 2\pi$, does $|\csc(\theta)| = 1$?

- A) $\frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2}$ C) $0, \frac{\pi}{2}, \frac{3\pi}{2}$ E) NOTA
 B) $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{2}$ D) $\frac{\pi}{2}, \frac{3\pi}{2}$

17) Given that $f(x) = \csc\left(\frac{1}{x}\right)$, find $f^{-1}(x)$.

- A) $\frac{1}{\arcsin\left(\frac{1}{x}\right)}$ B) $\frac{1}{\arccos\left(\frac{1}{x}\right)}$ C) $\arcsin\left(\frac{1}{x}\right)$ D) $\frac{1}{\sin(x)}$ E) NOTA

18) Determine the period of the function $g(x) = \sin(x) - \sin(2x) + \sin(3x) - \sin(4x) + \sin(5x)$.

- A) $\frac{\pi}{2}$ B) π C) 2π D) 3π E) NOTA

19) Find the domain of the function $g(x) = \ln(\arcsin(x))$

- A) $(0,1]$ B) $[0,1)$ C) $[1, \infty)$ D) $(-\infty,1]$ E) NOTA

20) One of the roots of $4x^2 + 48x + k = 0$ is three times the other. What is the value of k ?

- A) 54 B) 108 C) 162 D) 191 E) NOTA

21) When $x^9 + 1$ is factored to the form $(Ax+1)(Bx^2 - Cx + 1)(Ax^6 - Dx^3 + 1)$, find $A + B + C + D$.

- A) 0 B) 1 C) 3 D) 4 E) NOTA

22) Determine the value of k given that $\begin{vmatrix} 3 & 1 & 3 \\ k & 2 & -1 \\ 4 & 2 & -4 \end{vmatrix} = 2k + 3$

- A) $-\frac{8}{49}$ B) $-\frac{5}{49}$ C) $\frac{5}{49}$ D) $\frac{49}{8}$ E) NOTA

23) If $|5x+1| = 2 + |x-15|$ then determine the sum of all possible values of x .

- A) $\frac{-11}{6}$ B) $\frac{-6}{11}$ C) $\frac{6}{11}$ D) $\frac{43}{6}$ E) NOTA

24) Let $f(x+1) = 2x + 6$ and $f(p) = 18$, find the value of p .

- A) 5 B) 6 C) 7 D) 8 E) NOTA

25) Find the number of solutions to $4\cos(x)^2 - 2\cos(x) = 6$ on $x = [0, 2\pi]$.

- A) 0 B) 1 C) 2 D) 3 E) NOTA

26) Find the sum of all real x which satisfy $x^5 + 5x^4 - 10x^3 - 50x^2 + 9x + 45 = 0$.

- A) -5 B) -1 C) 1 D) 5 E) NOTA

27) Solve for all x : $\frac{|x-1|}{|x+7|} < x$

- A) $-4 - \sqrt{17} < x$ C) $4 - \sqrt{17} < x$ E) NOTA
B) $-4 + \sqrt{17} < x$ D) $4 + \sqrt{17} < x$

28) Find the sum of all real x which satisfy the following: $\frac{5}{5-x^7} = \frac{7}{7+x^7}$

- A) -7 B) -5 C) 0 D) $\frac{5}{7}$ E) NOTA

29) Find all real t such that $\frac{2|t|}{(|t|+1)(t+1)} = 1$.

- A) $-1 - \sqrt{2}$ C) $1 + \sqrt{2}$ E) NOTA
B) $-1 + \sqrt{2}$ D) $1 - \sqrt{2}$

30) What is the smallest possible value of $p(n) = 16n^2 + 15n - 5$ if n is restricted to integers.

- A) -5 B) -4 C) -1 D) 0 E) NOTA