

Alpha Applications
FAMAT State Convention 2003

For all questions, E. NOTA means none of the above answers is correct.

1. A person is dealt three random cards out of a standard deck of 52 cards. What is the probability of him getting dealt three cards of the same rank? (i.e. AAA)

A) $\frac{3}{850}$

D) $\frac{3}{1700}$

B) $\frac{1}{425}$

E) NOTA

C) $\frac{12}{5525}$

2. A spider crawls around the rim of a Frisbee with diameter 10 inches. How much distance does it travel after 67° around? (Answer to the nearest hundredth of an inch.)

A) 5.85

D) 335.00

B) 11.69

E) NOTA

C) 1.75

3. What is 103_4 represented as a binary number?

A) 11101

D) 10011

B) 10101

E) NOTA

C) 11001

4. For an angle ϕ , $\cot(\phi) = \frac{6}{\sqrt{2}}$ and $\sin(\phi) = -\sqrt{\frac{2}{38}}$. What is the value of $\cos(\phi)$?

A) $\frac{6}{\sqrt{38}}$

D) $-\frac{6}{\sqrt{2}}$

B) $\sqrt{\frac{2}{6}}$

E) NOTA

C) $\sqrt[3]{38}$

5. In the pictured parallelogram, $\angle BAD = 35^\circ$, $AB = 5$ and $BD = 13$.

What is the area of the parallelogram? (Figure not drawn to scale. Answer to the nearest hundredth.)

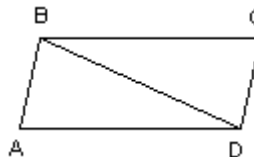
A) 50.00

D) 48.11

B) 42.56

E) NOTA

C) 38.92



6. What is the converse of the following statement?

“If the gators win today, they will be the champions.”

A) If the gators don't win today, they won't be champions.

B) If the gators win today, they will not be champions.

C) If the gators are champions, they will win today.

D) If the gators aren't champions, they won't win today.

E) NOTA

7. A triangle in space has vertices at the points (2,4,0) , (-5,7,0) and (12,3,3). What is the area of this triangle? (Answer to the nearest square unit.)

- A) 12.36
- B) 14.89
- C) 16.21
- D) 18.55
- E) NOTA

8. Given that $\sum_{n=1}^{\infty} \cos^n(\beta) = \frac{3}{2}$ and $0 \leq \beta \leq \frac{\pi}{2}$. Find $\sum_{n=1}^{\infty} \sin^n(\beta)$.

- A) 2
- B) 4
- C) 6
- D) -2
- E) NOTA

9. $X = \sqrt{9 - \sqrt{9 - \sqrt{9 - \sqrt{9 - \dots}}}}$ What is the value of X to the nearest hundredth?

- A) 2.54
- B) 2.61
- C) 2.65
- D) 3.0
- E) NOTA

10. For the following statements, A, B, and C are matrices.

- | | | |
|---|---|--|
| <ul style="list-style-type: none"> I. If $AB = AC$, then $B = C$ II. If A is $n \times n$, then $2A = 2^n A$ III. If A is $n \times n$, then $A^T = A$ | } | <ul style="list-style-type: none"> IV. A system of n equations and n unknowns has zero, one, or infinitely many solutions. V. If $A = 0$, then $A\bar{x} = \bar{0}$ has the only solution $\bar{x} = \bar{0}$ |
|---|---|--|

How many statements are true?

- A) 0
- B) 1
- C) 2
- D) 4
- E) NOTA

11. What is the value of x when the following algorithm completes?

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Input n from user;
x=0;
for j=1 to n do
{
    for k=1 to j do
    {
        x=x+1;
    }
}
    
```

- A) 1100
- B) $(n - 1)^2$
- C) $\frac{n(n + 1)}{2}$
- D) $n^2(n + 1)$
- E) NOTA

12. Evaluate $\begin{vmatrix} 5 & -7 & 2 & 2 \\ 0 & 3 & 0 & -4 \\ -5 & -8 & 0 & 3 \\ 0 & 5 & 0 & -6 \end{vmatrix} + \begin{vmatrix} 12 & 0 & 0 & 0 \\ x^2 & 2 & 0 & 0 \\ 20 & x & -1 & 0 \\ 11 & -55 & -5 & 1 \end{vmatrix}$. (Assume x is a real number)

- A) -4
B) 1
C) 4
D) $5x$
E) NOTA

13. Jason decides to go jogging along the graph of a third degree polynomial. He runs over the following points: $(-1,5)$, $(0,4)$, $(1,3)$ and $(2,8)$. Which one of the following coordinates will he eventually come to?

- A) $(3,10)$
B) $(3,25)$
C) $(4,55)$
D) $(4,70)$
E) NOTA

14. An integer between one and 15,328 (inclusive) is randomly chosen. What is the probability that this number is divisible by 17?

- A) $\frac{225}{3832}$
B) $\frac{901}{15328}$
C) $\frac{451}{7664}$
D) $\frac{111}{1916}$
E) NOTA

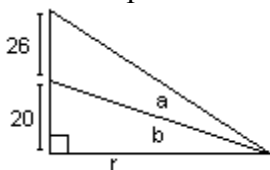
15. I have \$4.60 in quarters and dimes, with 28 total coins. How many quarters do I have?

- A) 17
B) 20
C) 16
D) 12
E) NOTA

16. What is the range of the function $f(x) = \frac{\pi x^2}{x^2 + 1}$?

- A) All real numbers
B) $(-\infty, -1) \cup (-1, \infty)$
C) $(0, \pi)$
D) $[-\pi, \pi]$
E) NOTA

17. In the pictured triangle, the two angles **a** and **b** are equal.



How long is the side **r** to the nearest tenth?

- A) 46.0
B) 48.9
C) 55.4
D) 56.3
E) NOTA

18. How many ways are there to paint 15 offices so that 4 are painted green, 5 are black, 3 are red, and the remaining ones white?

- A) 12612600
 B) 1848
 C) 5278
 D) 15!
 E) NOTA

19. A group contains 20 men and 20 women. How many distinct arrangements are there to sit these people in a row if the men and women must alternate seats?

- A) 2
 B) $(20!)^2$
 C) $\frac{(20!)^2}{2}$
 D) $2(20 \text{ nCr } 10)$
 E) NOTA

20. The time it takes for a specific algorithm to complete is determined by the size of its input, n . $T(n)$ is a function that returns the value of the time needed. Which of the following time functions is the fastest for large values of n ?

- A) $T(n) = n$
 B) $T(n) = n^2$
 C) $T(n) = \log n$
 D) $T(n) = n (\log n)$
 E) $T(n) = n!$

21. Which of the following is a correct recursive function for the n^{th} Fibonacci number?

- A) $f(n) = f(n-1) + f(n-2) + 1$ where $n > 1$
 $f(0) = 0$
 $f(1) = 1$
 B) $f(n) = f(n-1) + f(n-2)$ where $n > 1$
 $f(0) = 0$
 $f(1) = 1$
 C) $f(n) = 2f(n-1)$
 D) $f(n) = f(n+1) - f(n-2)$ where $n > 1$
 $f(0) = 0$
 $f(1) = 1$
 E) NOTA

22. A sailboat moves along a line such that its x and y coordinates at time $t > 0$ are $x = 2t$ and $y = t + 1$. At what time does the sailboat run into the island defined by $(x - 5)^2 + y^2 = 4$?

- A) $t = 0$
 B) $t = 2.5$
 C) $t = 3.75$
 D) Never
 E) NOTA

23. $\lim_{x \rightarrow \infty} \left(\frac{ax^2 + bx + c}{dx^2 + ex + f} \right) = 3$. What is the value of $\frac{d^2}{a^2}$?

- A) 3
 B) -3
 C) 9
 D) Cannot be determined
 E) NOTA

24. What is the area of a triangle with two sides of length 6 and an included angle of 65 degrees? (Answer to the nearest tenth.)

- A) 7.6
 B) 15.2
 C) 32.6
 D) 16.3
 E) NOTA

25. How far from the origin is the point $\frac{i}{i-1} + (i-1)^2$ when it is graphed in the complex number plane?

- A) $\sqrt{\frac{13}{2}}$
 B) $\frac{5}{2}$
 C) $\sqrt{\frac{5}{2}}$
 D) 0
 E) NOTA

26. The function $g(x)$ is defined for nonnegative integers such that:

$$g(x) = x \cdot g(x-1), \text{ for } x > 0$$

$$g(0) = 1$$

What is the value of $g(7)$?

- A) 720
 B) 5040
 C) 40320
 D) 1
 E) NOTA

27. An isosceles triangle has a perimeter of 17 and a base of length 5. What is the value of the vertex angle to the nearest hundredth of a degree?

- A) 49.25
 B) 40.75
 C) 156.44
 D) 81.21
 E) NOTA

28. What is the area of the parallelogram with vertices $(0,0)$, $(5,2)$, $(6,4)$, $(11,6)$?

- A) 4
 B) 2
 C) 8
 D) 66
 E) NOTA

29. There are 5 Smashing Pumpkins CDs, 4 Pearl Jam CDs, and 1 Sublime CD in Freddy's case. What is the probability that when Freddy randomly picks two CDs both will *not* be by the same band?

A) $\frac{9}{10}$

D) $\frac{16}{45}$

B) $\frac{4}{135}$

E) NOTA

C) $\frac{29}{45}$

30. $\log(a^2) - \log\left(\frac{a^2}{b^2}\right) - \log\left(\frac{b^2}{c^2}\right) - \log\left(\frac{c^2}{d^2}\right) = \log(50)$. What is the positive value of d to the nearest hundredth?

A) 7.07

D) 1.70

B) 50.00

E) NOTA

C) -7.07