#		Solution
1	C	$v=3i+2j+3k \Rightarrow \lVert v\rVert=\sqrt{9+4+9}=\sqrt{22}$
2	A	$v=3i+2j+3k \quad w=i+j+4k$ $\langle v,w\rangle=3\cdot1+2\cdot1+3\cdot3=14$
3	B	$\begin{vmatrix} i & j & k \\ 3 & 2 & 3 \\ 1 & 1 & 3 \end{vmatrix}=6i+3j+3k-(3i+9j+2k)$ $=3i-6j+k$
4	C	
5	D	
6	B	
7	A	$(-3,-1,2)$ and $(5,8,4)$ $v=(8,9,2)$ so the equation of the line is $(8,9,2)t+(-3,-1,2)$ which has the associated system of equations. $8t-3=x$ $9t-1=y$ the x-y plane has z=0 so $2t+2=z$ 2t+2=0 $\Rightarrow$ t = -1 so x = -11 and y = -10 point of intersection is $(-11,-10,0)$
8	D	A=(4,1,2) B=(1,5,4) C=(-3,2,6) v1 = (4-1, 1-5, 2-4) = (3,-4,-2) v2 = (4-(-3), 1-2, 2-6) = (7,-1,-4) a vector perpendicular to the plane is $\begin{vmatrix} i & j & k \\ 3 & -4 & -2 \\ 7 & -1 & -4 \end{vmatrix}=$ $16i-14j-3k-(2i-12j-28k)=$ $14i-2j+25k=v_3$ A general vector in the plane is v4 =(x-4, y-1, z-2) so the dot product of v4 and v3 is 0 $14\cdot(x-4)+-2(y-1)+25(z-2)=0$ $14x-56+-2y+2+25z-50=0$ $14x-2y+25z=56-2+50=104$ $14x-2y+25z=104$

#		Solution
9	C	F(x,y,z) = M **i** + N **j** + P **k** then the curl F is $\left(\frac{\partial P}{\partial y}-\frac{\partial N}{\partial z}\right)\mathbf{i}-\left(\frac{\partial P}{\partial x}-\frac{\partial M}{\partial z}\right)\mathbf{j}+\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)\mathbf{k}$ $\mathbf{F}(x,y,z)=xyz\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ (0 – 0) i – (0 – xy) j + (0 – xz) k at (1,2,1) = -(-2 j) + -1k = 2j – k
10	E	$f_{xy}=f_{yx}$ so the answer is 0 E
11	D	$z=2x^2+y^2 \quad 2x^2+y^2-z=0$ $f_x=4x \quad f_y=2y \quad f_z=-1$ at (1,1,3) $(4,2,-1)$ The equation is then $4(x-1)+2(y-1)-1(z-3)=0$ $z=4x+2y-3$ try points (1,1,3) 4+2-3 = 6-3 = 3 OK (2,3,11) 8+6-3 = 14-3 = 11 OK (2,-3,-1) 8-6-3 = 2-3 = -1 OK (3,-2,7) 12-4+7= 8+7=15 NO
12	C	if F = Mi + Nj + Pk $\frac{\partial M}{\partial x}+\frac{\partial N}{\partial y}+\frac{\partial P}{\partial z}$ div(2,1,-1) = $3x^2y^2z+0+0$ $=3x^2y^2z(2,1,-1)$ $=3\cdot4\cdot1\cdot-1=-12$
13	B	$w=yz^2-x=(2t-3)(1-t)^2-t^2$ $\frac{dw}{dt}=(2t-3)(2(1-t)(-1))+2(1-t)^2-t^2$ if t=1 then dw/dt = -2

#		Solution
14	A	If we move from (1,4) to (4,8) the direction vector is (4-1, 8-4) = (3,4) which is the legs of a 3-4-5 right triangle. Call the direction angle θ $\cos\theta = \frac{3}{5}$ and $\sin\theta = \frac{4}{5}$ $D_u f(x,y) = f_x \cos\theta + f_y \sin\theta$ $f_x = \frac{-x}{\sqrt{81-x^2-y^2}}$ so $f_x(1,4) = -\frac{1}{8}$ $f_y = \frac{-y}{\sqrt{81-x^2-y^2}} = -\frac{1}{2}$ $-\frac{1}{8}\cdot\frac{3}{5} + -\frac{1}{2}\cdot\frac{4}{5} = -\frac{3}{40} - \frac{2}{5} = -\frac{19}{40}$
15	A	(x,y) is a critical point of f if 1. fx = 0 AND fy=0 or 2. fx and fy do not exist $f_x = 2x - 6y$ $f_y = 2y - 6x$ Solving simultaneously $\Rightarrow$ (0,0)
16	A	$\int_0^4 \int_y^2 \left(y^2 - xy\right) dxdy$ $y^2 x - \frac{1}{2}x^2 y \Big\|_y^2 = 2y^2 - 2y - \left(y^3 - \frac{1}{2}y^3\right)$ $= 2y^2 - 2y - \frac{1}{2}y^3$ $\frac{2}{3}y^3 - y^2 - \frac{1}{8}y^4 \Big\|_0^4 = \frac{128}{3} - 16 - \frac{1}{8}\cdot 256$ $= \frac{128}{3} - 16 - 32 = \frac{128}{3} - 48 = -\frac{16}{3}$
17	D	Orthogonal means perpendicular so take the dot product with each choice and find the one with an answer different from 0 (2,-1,3)*(1,-1,-1) = 2+1-3=0 No (2,-1,3)*(1,2,0) = 2-2=0 No (2,-1,3)*(3,0,-2) = 6-6=0 No (2,-1,3)*(1,3,-1) = 2-3-3= -4 NOT ORTHOG
18	B	The magnitude of the cross product of the two vectors is the AREA of the parallelogram formed by the two vectors. The area of the triangle is 1/2 of that. pts (0,0,1) (0,2,0) and (3,0,0) v1 = (3,0,0)-(0,0,1) = (3,0,-1) v2 = (3,0,0)-(0,2,0) = (3,-2,0) $\begin{vmatrix} i & j & k \\ 3 & 0 & -1 \\ 3 & -2 & 0 \end{vmatrix} = \left\Vert(2i - 3j - 6k)\right\Vert = 7$ so area is 7/2
19	C	$D = \frac{\left\vert ax_0 + by_0 + cz_0 + d\right\vert}{\sqrt{a^2+b^2+c^2}}$ $D = \frac{\left\vert 1\cdot 0 + 2\cdot 0 + 2\cdot 0 + 6\right\vert}{\sqrt{1^2+2^2+2^2}} = \frac{6}{\sqrt{9}} = 2$
20	A	partial derivatives expressions can't be "cancelled" like we do with the chain rule for single variables. To calculate the three partial derivatives first solve for the 3 variables. PV=nRT $P = \frac{nRT}{V} = nRTV^{-1} \Leftrightarrow$ $\frac{\partial P}{\partial V} = nRT\left(-1V^{-2}\right) = \frac{-nRT}{V^2}$ $V = \frac{nRT}{P} \Rightarrow \frac{\partial V}{\partial T} = \frac{nR}{P}$ $T = \frac{PV}{nR} \Rightarrow \frac{\partial T}{\partial P} = \frac{V}{nR}$ $\frac{-nRT}{V^2}\cdot\frac{nR}{P}\cdot\frac{V}{nR} = \frac{-nRT}{VP} = \frac{-nRT}{\frac{nRT}{P}\cdot P} = -1$

#		Solution
21	D	differentiate implicitly find dz/dy $x^3z^5 - y^2z^3 - 3xy = 1$ $x^3\left(5z^4\right)\frac{\partial z}{\partial y} - \left(2yz^3 + y^2\left(3z^2\frac{\partial z}{\partial y}\right)\right) - 3x = 0$ $\frac{\partial z}{\partial y} = \frac{3x + 2yz^3}{5x^3z^4 - 3y^2z^2}$ we need to solve for z so substitue (-1,1) into the original equation $z^5 + z^3 - 2 = 0$ 1 is a solution $\Rightarrow$ $\frac{\partial z}{\partial y} = \frac{3(-1) + 2(1)(1)^3}{5(-1)^3(1)^4 - 3(1)^2(1)^2} = \frac{-1}{-5-3} = \frac{1}{8}$
22	D	f(x,y)= $x^3 - y^2 + 27$ To evaluate a line integral over a path you must parametrize the path and change the form of the integral $\int_C f(x,y)\,ds =$ $\int_a^b f(x(t),y(t))\sqrt{(x'(t))^2 + (y'(t))^2}\,dt$ parametrization is x = t y = $t^{\frac{3}{2}}$ $f(x(t),y(t)) = t^3 - \left(t^{\frac{3}{2}}\right)^2 + 27 = 27$ x'(t) = 1 $y'(t) = \frac{3}{2}t^{\frac{1}{2}}$ $\int_0^1 27\sqrt{1 + \frac{9}{4}t}\,dt = 27\int_0^1 \frac{1}{2}\sqrt{4+9t}\,dt$ u = 4 + 9t du = 9 dt substituted integral is $\frac{27}{2\cdot 9}\int_4^{13} u^{\frac{1}{2}}du = \frac{3}{2}\cdot\frac{2}{3}u^{\frac{3}{2}}\Big\|_4^{13} = \left(13^{\frac{3}{2}} - 4^{\frac{3}{2}}\right)$ $= 13\sqrt{13} - 8$
23	C	$\int_0^4\int_0^1\int_0^x dydxdz$ $\int_0^4\int_0^1\left(y\,\Big\|_0^x = x\right)dxdz = \int_0^4\int_0^1 xdxdz$ $\int_0^4\left(\frac{1}{2}x^2\,\Big\|_0^1\right)dz = \int_0^4\frac{1}{2}dz$ $= \frac{1}{2}z\,\Big\|_0^4 = 2$
24	D	$\\|(2,2,3)\\| = \sqrt{2^2 + 2^2 + 3^2} = \sqrt{17}$
25	C	$\frac{(2,2,1)}{\\|2,2,1\\|} = \frac{(2,2,1)}{\sqrt{4+4+1}} = \left(\frac{2}{3},\frac{2}{3},\frac{1}{3}\right)$
26	C	Find critical points by looking at first partial derivatives $f_x = 2x + 2$ $f_y = 2y - 2$ for fx and fy to be zero the critical point must be (-1,1) To check for max/min calculate $d = f_{xx}f_{yy} - \left(f_{xy}\right)^2$ if d>0 and fxx > 0 then rel min if d>0 and fxx < 0 then rel max if d<0 then saddle point inconclusive if d=0 $f_{xx} = 2$ $f_{yy} = 2$ $f_{xy} = 0$ so d = 4 therefore there's a rel minimum at (-1,1) Substitute into the original equation 1 + 1 –2 – 2 = -2
27	B	The norm of a partition is the length of the longest diagonal in all the rectangles made when the plane is partitioned. Since this is a regular partition, we only need to check one diagonal. $\sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{5}{16}} = \frac{\sqrt{5}}{4}$
28	A	$A = ab\sin x$ $\frac{\partial A}{\partial a} = b\sin x \Rightarrow 20\sin\frac{\pi}{6} = 10$

#		Solution
29	B	check point on surface 3-1+2+3-7=0 OK $F_x = 6xz - 2xy^2$ so at $(1,1,1)$ $\to 6\cdot 1\cdot 1 - 2\cdot 1\cdot 1 = 4$ $F_z = 3x^2 + 6z^2 + 3y$ so $\to 3\cdot 1 + 6\cdot 1 + 3\cdot 1 = 12$ $\frac{\partial z}{\partial x} = -\frac{F_x(1,1,1)}{F_z(1,1,1)} = -\frac{1}{3}$
30	B	$f_x = \frac{y}{x} + y^2 \Rightarrow \frac{2}{1} + 2^2 = 6$ $f_y = \ln x + 2xy \Rightarrow \ln 1 + 2\cdot 1\cdot 2 = 4$ $\nabla f(1,2) = (6,4)$