2003 FAMAT STATE CONVENTION
THETA INDIVIDUAL SOLUTIONS

1

$2x + 10 = 70$

$2x = 60$

$x = 30$

C

2

$\text{slope} = \frac{4}{-(-2)} = 2$

D

3

$= \sum_{n=1}^{5} (n+1)^2$

$= 2^2 + 3^2 + 4^2 + 5^2 + 6^2$

$= 90$ E

4

$f(g(x)) = \left(\sqrt[4]{x-5}\right)^4 + 5$

$x - 5 \geq 0$

$x \geq 5$

$[5, \infty)$ C

5

$\Rightarrow 2\log_a x + 3\log_a x + \ldots + 10\log_a x = 54$

$54 \log_a x = 54$

$\log_a x = 1$

A

6

$T = \frac{4}{11}(83+16)^2$

$= 3564$

D

7

$75 = 5^2 \cdot 3$

$80 = 2^4 \cdot 5$

$90 = 2 \cdot 5 \cdot 3^2$

$LCM = 2^4 \cdot 3^2 \cdot 5^2 = 3600$

E

8

$x^1y^1z^8 + 2x^4y^5 - 6x^2y^3z^2$

$1+1+8 \quad 4+5 \quad 2+3+2$

$10 \quad 9 \quad 7$

degree $= 10$ D

9

✓ I closed

II not closed (division by zero)

✓ III closed (smallest value $= 1 \cdot 3 =$

I, III closed C

10

$(5i)^2 + 1 - (9 - 16i^2)$

$-25 + 1 - 9 - 16$

$= -49$ A

11

$m = \frac{-15-9}{12-3} = \frac{-24}{9} = \frac{-8}{3}$

$y = mx + b$, plug in m, $(3, 9)$

$9 = -\frac{8}{3}(3) + b$

$b = 17$ C

12

arith: 4 _ _ _ 5184

$5184 - 4 = 5180 \quad \frac{5180}{4} = 1295$

add to get next terms (largest)

4, 1299, 2594, (3889), 5184

geom: common difference $= \pm 6$

4, ± 24, 144, ± 864, 5184

largest either 144 or 864

$3889 + 144 = 4033$ ✓

$3889 + 864 = 4753$

4033 D

13

yellow die	yellow score	red score needed	red combinations
1	3	13	—
2	6	10	6-4, 5-5, 4-6
3	9	7	1-6, 2-5, 3-4, 4-3, 5-2, 6-1
4	12	4	1-3, 2-2, 3-1
5	15	1	—
6	18	-2	—

A

14

$G - 6 = \frac{7}{9}(W-6) \Rightarrow 9G - 54 = 7W - 42$

$G + 9 = \frac{6}{7}(W+9) \Rightarrow 7G + 63 = 6W + 54$

$6(9G - 7W = 12) \quad 54G - 42W = 72$

$-7(7G - 6W = -9) \quad -49G + 42W = 63$

$5G = 135$

$G = 27$

$W = 33$

$27 \cdot 33 = 891$ D

15

If $a^b = 1$, $a = 1$ or $b = 0$

$x^2 - 2x - 7 = 1 \qquad x^2 - 9x + 20 = 0$

$x^2 - 2x - 8 = 0 \qquad (x-4)(x-5) = 0$

$(x-4)(x+2) = 0 \qquad x = 4, 5$

$x = 4, -2$

solutions $-2, 4, 5$ $\quad -2+4+5 = 7$ C

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16 $y = \frac{4x-3}{5x+2}$

$\frac{4}{5}$ is restricted from the range **D**

17 $(\sqrt{18+\sqrt{308}})^2 = (\sqrt{A}+\sqrt{B})^2$

$18+2\sqrt{77} = A+B+2\sqrt{AB}$ → $A+B=18$, $AB=77$

$A=7$, $B=11$

$\Rightarrow \sqrt{7}+\sqrt{11}$

By the same method, $\sqrt{20+\sqrt{336}} = \sqrt{14}+\sqrt{6}$

$\frac{\sqrt{7}+\sqrt{11}}{\sqrt{6}+\sqrt{14}} \cdot \frac{(\sqrt{6}-\sqrt{14})}{(\sqrt{6}-\sqrt{14})} = \frac{\sqrt{42}+\sqrt{66}-\sqrt{98}-\sqrt{154}}{6-4}$

$= \frac{\sqrt{42}+\sqrt{66}-\sqrt{154}-7\sqrt{2}}{-8}$

$T+U+V = 42+66+154 = 262$ **D**

18 $y = \frac{4x^2-7}{9-x^2}$

vertical: $9-x^2=0$

$x = \pm 3$

horizontal: $y = \frac{4}{-1}$

$y = -4$ **D**

19 Betsy $A = P(1+\frac{r}{n})^{nt}$

$= 3000(1+\frac{.025}{4})^{4\cdot 4}$

$= 3314.48$

Leesa $A = Pe^{rt}$

$= 3000\,e^{.025(4)}$

$= 3315.51$

difference \$1.03 **B**

20 For $n>3$ and even integers, a polygon with n sides has

$\frac{n}{2}$ longest diagonals (opposite vertices)

total diagonals $= \frac{n(n-3)}{2}$

$\text{Prob} = \frac{\text{favorable}}{\text{total}} = \frac{\frac{n}{2}}{\frac{n(n-3)}{2}} = \frac{1}{n-3}$ **D**

21 $\log 6^{x-3} = \log 4^{x+5}$

$(x-3)\log 6 = (x+5)\log 4$

$x\log 6 - 3\log 6 = x\log 4 + 5\log$

$x(\log 6 - \log 4) = \frac{5\log 4 + 3\log 6}{\log 6 - \log 4}$

$x = 30.3521806\ldots$ 6th digit is 0 **A**

22 $f(-1) = -14+3+6-3$

$= -8$

B

23 $n = k+1$

A

24 $M+N+P = 12$ (#1)

$4M+2N+P = 25$ (#2)

$9M+3N+P = 44$ (#3)

subtract #1 and 2 $\Rightarrow -3M-N = -13$ (

subtract #2 and 3 $\Rightarrow -5M-N = -19$ (

subtract #4 and 5 $\Rightarrow 2M = 6 \Rightarrow M =$

plug into #4 $\Rightarrow N = 4$

plug into #1 $\Rightarrow P = 5$

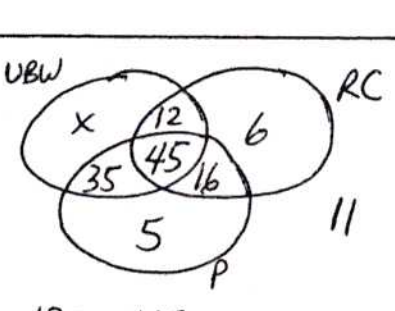
$M(N)(P) = 3(4)(5) =$ 6

D

25 $Z^{-1} = \begin{bmatrix} -\frac{5}{9} & -\frac{1}{9} & \frac{1}{3} \\ -\frac{1}{9} & \frac{2}{9} & -\frac{2}{3} \\ -\frac{2}{9} & \frac{4}{9} & -\frac{1}{3} \end{bmatrix}$

Sum of elements $= \frac{1}{9}$ **A**

26



$x + 130 = 139$

$x = 9$ **C**

27 $f(x-1) = (x-1)^3 + 3(x-1) + 4$

$= x^3 - 3x^2 + 3x - 1 + 3x - 3 + 4$

$= x^3 - 3x^2 + 6x$

$= x(x^2 - 3x + 6)$

D

28 $(x^2+1)^2 - 7x^2 + 5 = 0$

$(x^2+1)^2 - 7x^2 - 7 + 12 = 0$

$(x^2+1)^2 - 7(x^2+1) + 12 = 0$

$(x^2+1-4)(x^2+1-3) = 0$

$x^2 = 3 \quad x^2 = 2$

$x = \pm 3, \pm 2$ $\quad |\sqrt{3}| + |-\sqrt{3}| + |\sqrt{2}| + |-\sqrt{2}|$

$= 6.29252874$

29

hyperbola

C

30 $\frac{3^{3(x+2)} \cdot 3^{2(2x-5)} \cdot 3^{x}}{3^{4(3x-2)} \cdot 3^{5(5x+3)}} = 3^{y}$

$\frac{3^{8x-4}}{3^{37x+7}} = 3^{-29x-11} = 3^{y}$

$y = -29x - 11$ $\quad -29 - 11 = -40$

A