THETA APPLICATIONS
FAMAT STATE CONVENTION 2003
SOLUTIONS

1. $t/4 + t/7 - t/10 = .80-.25 = .55$; $70t + 40t - 28t = 154$; $82t = 154$; $t = 1.878$ hrs (B)
2. Two geometric infinite series: down $10 + 5 + 2.5 + \ldots = 10/(1 - .5) = 20$; up $5 + 2.5 + 1.25 + \ldots = 10$; $20 + 10 = 30$ ft (B)
3. Time up stream $25/(r-4)$; time down stream $25/(r+4)$; $25/(r-4) + 25/(r+4) = 6$; $25(r+4) + 25(r-4) = 6(r^2-16)$; $0 = 6r^2 - 50r - 96$; $r = (25 \pm \sqrt{1201})/6$; only $r = (25 + \sqrt{1201})/6$ is possible since it is >0. (D)
4. Equilibrium occurs when $D = S$; $900 - 20P = 175 + 5P$; $P = 29$; $900 - 20(29) = 320$ (C)
5. (M,F), (25,40), (31,48); $m = 4/3$; $F - 40 = 4/3\ (M-25)$; $F = (4/3)M - 100/3 + 40 = (4/3)M + 20/3$ (E)
6. $80 \leq 0.60$(test avg) $+ 0.20$(quiz avg) $+ 0.20$(final) < 90; test avg $= (75+80+90+100+92)/15 = 87.4$; quiz avg$= (80+90+100+95+78+84+93)/7 = 620/7$; $80 \leq .6(87.4) + .2(620/7) + .2(\text{final}) < 90$; $80 \leq 52.44 + 17.714 + .2(\text{final}) < 90$; $9.846 \leq .2(\text{final}) < 19.846$; $49.23 \leq \text{final} < 99.23$; [50,99] (D)
7. $A = 5000(1 + 0.125/4)^{(4*13)} = \$24768.29$ nearest penny (D)
8. 7 lb $= 16(7) = 112$ oz; $75/100 \leq 112/(x + 112) \leq 90/100$; $.75(x+112) \leq 112$ and $112 \leq .90(x + 112)$; $.75x \leq 28$ and $11.2 \leq .90x$; $x \leq 37\ 1/3$ and $12\ 4/9 \leq x$; [12 4/9,37 1/3] (C)
9. $300(10)=3000$; $(300 - 15x)(10+x) = 3000 + 150x - 15x^2 = -15(x^2-10x)+3000$; as a parabola: $y-3000 = -15(x^2-10x)$; $y-3375 = -15(x^2-10x+25) = -15(x-5)^2$; vertex is (5,3375); fare is $10 + 5 = 15$ (B)
10. Abel: $(x-6)(x+3)=x^2-3x-18$; Cain: $(x+5)(x+3)=x^2+8x+15$; original must be $x^2+8x-18$; $x = -4 \pm \sqrt{34}$ (C)
11. (Distance)(Weight) on the left side= (Distance)(Weight) on the right; $75(12-x)=300x$; $900-75x=300x$; $2.4=x$ (A)
12. $Y=-5t^2+985t+3000$; $y-3000 = -5(t^2-197t)$; $y-3000-48511.25=-5(t^2-197t+9702.25)$; $y-51511.25=-5(t-98.5)^2$; vertex is (98.5,51511.25) height is 51511.25 ft (D)
13. Law of Cosines: $x^2 = 225^2 + 200^2 - 2(225)(200)\cos(135°)$; $x = \sqrt{154264.610} = 392.765$; 393 (D)
14. $500e^{(0.015t)} = .40(500) + 500 = 700$; $e^{(0.015t)} = 1.4$; $0.015t = \ln(1.4)$; $t = \ln(1.4)/0.015 = 22.431$ hr (B)
15. X represents the boots; Y represents the sandals; $10 \leq X \leq 80$, graph the two vertical lines; $Y \leq 150$, graph the horizontal line; $X+Y \leq 180$, graph the line and shade the inequalities; $P=115X + 16Y$, check the vertices of the shaded region for maximum profit; (10,150), P=3550; (30,150), P=5850; (80,100), P=10800 (E)
16. The system of equations: $X + Y + Z = 3700$; $X + Y = 2200$; $X + Z = 2400$; combine the first and third $Y = 1300$; $X + 1300 = 2200$, so $X = 900$; $900 + Z = 2400$, so $Z = 1500$; $Z - Y = 1500 - 1300 = 200$ (A)
17. Using a Venn diagram: rectangle U is the universal, three overlapping circles, A, B, and C, A and B overlap, as B and C, and as A and C, there must be an intersection of the three. Place 120 in B outside the overlaps. Place 30 in the three-way intersection. Place 25 in the intersection of A and C, 42 in the intersection of B and C, and 18 in the intersection of A and B, each outside the region with the 30. Place 150 in A

outside the overlaps, and 110 in C also outside the overlaps. This leaves 5 outside all the regions and in the universal set. 30 + 5 = 35 (C)

18. V=kT/P; 231=42k/20; k=110; V=110(30)/15 = 220 (D)
19. $(5+x)^2 + (12-x)^2 = 13^2$; $x^2+10x+25 +x^2-24x+144=169$; $2x^2-14x=0$; x=0 or 7; since the ladder moved x = 7 (E)
20. 140/x + 15 = 140/(x-3); 140(x-3) +15x(x-3) = 140x; $140x-420+15x^2-45x-140x =0$; $15x^2-45x-420=0$; x = -4 or 7, must be 7 (B)
21. $r^2=20L$; $r= \sqrt{(20)(275)} = 74.162$ (D)
22. 175000 – .05(175000) = 166250; I = 166250(.07125)(30)=355359.375; 166250 + 355359.375=521609.375; 521609.375/360=1448.91 rounded; (3300+975)/12= 356.25; 356.25+1448.91= 1805.16 (C)
23. Similar triangles: ht/base (x+16)/(2x-17) = (x+1)/(x-7); (x-7)(x+16)=(2x-17)(x+1); $x^2+9x-112=2x^2-15x-17$; $0=x^2-24x+95$; x=19 or 5, must be 19; ratio is 35/20 or 7/4 (E)
24. BL = blue from left pocket, BR = blue from right pocket, WL = white from left pocket; P(BR)= P(BL∩BR) + P(WL∩BR)= P(BL)·P(BR | BL) + P(WL)·P(BR | WL) = (5/9)(5/10) + (4/9)(4/10) = 41/90 (D)
25. 9!/(3!·4!·2!) = 1260 (B)
26. $4x^2-y^2=7$ and $2x^2+5y^2=9$; $4x^2-y^2=7$ and $-4x^2-10y^2=-18$; y= ±1, x= $\pm\sqrt{2}$; A=$(2\sqrt{2})2$ = $4\sqrt{2}$ (C)
27. Each lot is x in length and y in width. 6x+6y=600; y=100-x; A= 2x•2(100-x); $-4(x^2-100x)=A$; $-4(x^2-100x+2500)=A-10000$; $-4(x-50)^2=A-10000$; vertex (50,10000); lot is 50by50; plot is 100by100 (B)
28. The system of equations: n + d + q = 49, 0.05n + .10d + .25q = 6.45, 2n+3d+q=103; n=14 nickels, d=20 dimes, and q=15 quarters; 15-14=1 (A)
29. With the wind the rate is r + w = 33/1.5 = 22; Against the wind it is r–w=33/4.125=8; 2r=30; r=15; wind (w) is 7 (A)
30. $(x+8)^2+(x-8)^2=(x\sqrt{2}+2\sqrt{2})^2$; $x^2+16x+64 + x^2-16x+64= 2x^2+8x+8$; $2x^2+128= 2x^2+8x+8$; 120=8x; x=15 (B)