Logs & Exponents - Alpha - Answers

FAMAT State Convention - 2003

1. $\log_{12} 35 = \dfrac{\log 35}{\log 12} \approx 1.4308 \to B.$

2. $\log_b \sqrt[5]{a} = \frac{1}{5}\log_b a = \dfrac{13.5}{5} \to D$

3. On the given number line, AB = $\log_b y - \log_b x = \log_b \dfrac{y}{x}$

   CD = $\log_b y^3 - \log_b x^3 = \log_b \dfrac{y^3}{x^3} = \log_b\left(\dfrac{y}{x}\right)^3 = 3\log_b\left(\dfrac{y}{x}\right) = 3\text{AB} \to B$

4. $\ln(\sin x) - \ln(\cos x) = 1 \Rightarrow \ln\left(\dfrac{\sin x}{\cos x}\right) = 1 \Rightarrow$

   $\ln(\tan x) = 1 \Rightarrow e^1 = \tan x \Rightarrow x = \arctan e \to A$

5. $43^{-1} = .\underline{023255813953488372093}023255813953488372093023255813953488372093\ldots$
   (period underlined) therefore $\to C$

6. $\begin{cases} \log_x 9 + \log_8 y = \dfrac{7}{3} & let \log_x 9 = a \Rightarrow \log_9 x = \dfrac{1}{a} \\ \log_9 x + \log_y 8 = \dfrac{7}{2} & let \log_y 8 = b \Rightarrow \log_8 y = \dfrac{1}{b} \end{cases}$ that gives $\begin{cases} a + \dfrac{1}{b} = \dfrac{7}{3} \\ \dfrac{1}{a} + b = \dfrac{7}{2} \end{cases}$ $a = \dfrac{1}{3}, b = \dfrac{1}{2}$ or $a = 2, b = 3$

   Has solutions (729,64) and (3,2) therefore $\to D$

7. $\log_b \sqrt[4]{t^3 b^2} = \log_b t^{\frac{3}{4}} b^{\frac{2}{4}} = \log_b t^{\frac{3}{4}} + \log_b b^{\frac{2}{4}} = \frac{3}{4}\log_b t + \frac{1}{2} = \frac{3}{4}(8.64) + \frac{1}{2} = 6.98 \to D$

8. $\log(2^{10000}) = 10000 \times \log 2 \Rightarrow 2^{10000}$ is a 3010 digit number.

   $\log(2^{1000}) = 1000 \times \log 2 \Rightarrow 2^{1000}$ is a 301 digit number. therefore $\to A$

9. $2003^{-2003} = 10^{-2003\log 2003} = 10^{-6613.26694144} =$

   $10^{-.26694144} \times 10^{-6613} = .5408 \times 10^{-6613} = 5.408 \times 10^{-6614} \to B$

10. $2003^{2003} = 10^{2003\log 2003} = 7^{\frac{2003\log 2003}{\log 7}} = 7^{7825.44347319} = 7^{.44347319} \times 7^{7825} \to D$

11. An exponential function has a horizontal asymptote, therefore $\to E$

12. $(350 - T) = T_0 b^t;\ T_0 = 305;\ (350 - 71) = 305b^{.25} \Rightarrow b = .700192501821$
    $(350 - 185) = 305(.700292501821)^t \Rightarrow t = 1.7238112757 \to C$

13. $\log_b 25 = \log_b 5^2 = 2\log_b 5 = \frac{2}{3}\log_b 5^3 = \frac{2}{3}c \to D$

14. $x^2 \log 2 + x = \log 4096 + x\log 5 \Rightarrow x^2 \log 2 + x - x\log 5 = \log 4096$
    $x^2 \log 2 + x(1 - \log 5) - \log 2^{12} = 0 \Rightarrow x^2 \log 2 + x\log 2 - 12\log 2 = 0$
    $x^2 + x - 12 = 0 \Rightarrow x \in \{-4, 3\} \Rightarrow a + b = -1 \to B$

15. $4000^{-4000} = 10^{-4000 \log 4000} = 10^{-14408.2399653} = 10^{-.2399653} \times 10^{-14408} \rightarrow C$

16. $\log x = 2 \Rightarrow x = 100$. $100 \div 2\pi = 15.91\ldots$ In the first quadrant, the log graph will intersect the first period only once and then twice for the other 15 cycles until it achieves a height of 2. on the negative side of the y-axis, the log function will intersect the sin graph once in the fourth quadrant and then twice for each of the 16 cycles until it achieves a height of 2. Therefore $\rightarrow C$

17. $\dfrac{1}{\log_{b^2} 10} = \log_{10} b^2 (B) = 2\log_{10} b(D) = -2\log_{10} \frac{1}{b} \rightarrow C$

$$\log_6(3x-1) = \log_6\left(\frac{1}{x+2}\right) + 1 \Rightarrow \log_6(3x-1) - \log_6\left(\frac{1}{x+2}\right) = 1 \Rightarrow$$

18. $\log_6[(3x-1)(x+2)] = 1 \Rightarrow 6^1 = (3x-1)(x+2) \Rightarrow 3x^2 + 5x - 2 = 6 \Rightarrow$

$3x^2 + 5x - 8 = 0 \Rightarrow x \in \{\frac{-8}{3}, 1\}$; since $\log_6(3x-1)$ is undefined when $x = \frac{-8}{3} \therefore \rightarrow A$

19. $x^y + x^y = 2x^y = x^y \times x^1 \Rightarrow x = 2 \rightarrow B$

$$\log_b(22!) = \log_b\left(2^{19}3^9 5^4 7^3 11^2 \times 13 \times 17 \times 19\right) =$$

20. $\log_b 2^{19} + \log_b 3^9 + \log_b 5^4 + \log_b 7^3 + \log_b 11^2 + \log_b 13 + \log_b 17 + \log_b 19 =$

$19\log_b 2 + 9\log_b 3 + 4\log_b 5 + 3\log_b 7 + 2\log_b 11 + \log_b 13 + \log_b 17 + \log_b 19 =$

$19d + 9e + 4f + 3g + 2h + i + j + k \rightarrow E$

21. $2 = \left(1 + \frac{.0525}{12}\right)^{12t} \Rightarrow t \approx 13.23 years \rightarrow D$

22. 2003! has 997 factors of 3 and 3998 factors of 2 (1999 factors of 4) therefore there would be 997 zeros at the end of 2003! $\rightarrow A$

23. $\log 17 \times \dfrac{1}{\log e} = \ln 17$, since $\dfrac{1}{\log e} = \ln 10 \rightarrow A$

24. $e^k = \left(\dfrac{1}{2}\right)^{\frac{x}{2}} \Rightarrow e^k = (e)^{\frac{-x}{2}\ln 2} \Rightarrow k = \dfrac{-x}{2}\ln 2 \Rightarrow x = \dfrac{-2k}{\ln 2} = -2k\log_2 e \rightarrow B$

25. $\dfrac{1}{\log_4 18} + \dfrac{1}{2\log_6 3 + \log_6 2} + \dfrac{5}{\log_3 18} = \log_{18} 4 + \dfrac{1}{\log_6 18} + 5\log_{18} 3 =$

$\log_{18} 2^2 + \log_{18}(2^2 3) + \log_{18} 3^5 = \log_{18}(2^3 3^6) = \log_{18}(2 \times 3^2)^3 = 3 \rightarrow C$

26. If $x^{x^{x^{\cdot^{\cdot}}}} = 2$ then $x^2 = 2 \Rightarrow x = \sqrt{2} \rightarrow B$

27. $f_1(x) > f_3(x) > f_2(x) > f_4(x) \rightarrow B$

28. $\dfrac{10^{8.5}}{10^7} \approx 31.6 \rightarrow D$

29. $82.3 = 51.4 + 6.1 \times 2.3^{3.4x}. \Rightarrow x = .572927207447\ldots \rightarrow A$

30. $81^{\log_9 19} = \left(9^2\right)^{\log_9 19} = \left(9^{\log_9 19}\right)^2 = 19^2 = 361 \rightarrow D$