1) $\int_{\frac{\pi}{4}}^{\frac{2\pi}{3}} \sin(2x)dx = \left.\frac{-\cos 2x}{2}\right|_{\frac{\pi}{4}}^{\frac{2\pi}{3}} = \frac{-\cos 2(2\pi/3)}{2} - \frac{-\cos 2(\pi/4)}{2} = \frac{1}{4}$ B

2) Area= $\int_0^3 10-(10-2^x) = \int_0^3 2^x = \left.\frac{2^x}{\ln 2}\right|_0^3 = \frac{7}{\ln 2}$ D

3) $2x^2-3x = |2x^2-3x|$ for $[-\infty,0]\cup\left[\frac{3}{2},\infty\right]$ and $-(2x^2-3x) = |2x^2-3x|$ for $\left[0,\frac{3}{2}\right]$

$\int_{-1}^{\frac{5}{2}} |2x^2-3x|dx = \int_{-1}^{0} 2x^2-3xdx + \left|\int_0^{\frac{3}{2}} 2x^2-3xdx\right| + \int_{\frac{3}{2}}^{\frac{5}{2}} 2x^2-3xdx = \frac{131}{24}$ A

4) $\frac{1}{2-0}\int_0^2 \frac{x^2-6x-3}{x^2-6x+9} = \left.\frac{x^2+x}{2x-6}\right|_0^2 = -3$ B

5) $\int \theta\sqrt{1+\theta^2}d\theta = \frac{\sqrt{(\theta^2+1)^3}}{3}+C$ D

6) $\int \frac{-e^{-x}}{1-e^{-x}}dx = \ln(e^x+1)-x+C$ C

7) $\int_1^2 \frac{6}{x^2}dx = f(c)(2-1) \Rightarrow f(c)=3 \Rightarrow c=\sqrt{2}$ A

8) $\pi\int_1^3 \left(6^2-(6+x-x^2)^2\right)dx = \pi\int_1^3 (-x^4+2x^3+11x^2-12x)dx$ D

9) $\int \frac{dx}{x^2-2x+3} = \int \frac{dx}{(x-1)^2+2} = \frac{1}{\sqrt{2}}\arctan\frac{x-1}{\sqrt{2}}+C = \frac{\sqrt{2}}{2}\arctan\frac{\sqrt{2}(x-1)}{2}+C$ B

10) Area = $\int_0^{2\pi} 4\sin\frac{x}{2} - 2\sin xdx = 16$ E

11) $\int \frac{\csc z}{\cot^2 z}dz = \int \frac{\sin z}{\cos^2 z}dz = \int \tan z\sec zdz = \sec z + C$ B

12) $x^2+y^2-4x+6y=3 \Rightarrow (x-2)^2+(y+3)^2=16 \Rightarrow$ r=4, center=(2, -3) Volume = $(4^2\pi)(2\pi(9-2)) = 224\pi^2$ C

13) $\int 4^{2x}dx = \int 16^x dx = \frac{16^x}{4\ln 2}+C$ D

14) $\int \frac{4\arctan 2x}{4x^2+1}dx \Rightarrow u=\arctan 2x$, $du = \frac{2}{4x^2+1}dx \Rightarrow 2\int udu = u^2+C = \arctan^2 2x + C$ A

15) $\int_1^4 (4ax^2+2x+3a)dx = 93a+15$, $\int_{-2}^1 (2ax^2-3ax+2)dx = \frac{21a}{2}+6 \Rightarrow\ 93a+15 = \frac{21a}{2}+6 \Rightarrow\ a = -\frac{6}{55}$ C

16) Shell method: $2\pi\int_{\frac{7}{2}}^4 \frac{x}{(x-3)^2}dx = 2\pi\int_{\frac{7}{2}}^4 \left[\frac{1}{x-3}+\frac{3}{(x-3)^2}\right]dx = \frac{\pi(8\ln 2+9)}{4}$ B

17) $\pi\int_0^{\frac{4}{3}}(4-3x)^2\,dx=\frac{64\pi}{9}$ or simply using the volume formula $v=\frac{\pi r^2 h}{3}=\frac{\pi(4)^2(4/3)}{3}=\frac{64\pi}{9}$ E

18) $\int 3x(3-x^2)^3dx$, $u=3-x^2$, $du=-2xdx\Rightarrow -\frac{3}{2}\int u^3du=\frac{u^4}{4}+C=\frac{-3(3-x^2)^4}{2(4)}=\frac{-3(3-x^2)^4}{8}+C$ D

19) Area= $\int_2^8(2x-9)-(x^2-8x+7)dx=36$, $\bar{x}=\frac{1}{36}\int_2^8 x[(2x-9)-(x^2-8x+7)]dx=5$,

$\bar{y}=\frac{1}{36}\int_2^8\left[\frac{(2x-9)+(x^2-8x+7)}{2}\right][(2x-9)-(x^2-8x+7)]dx=-\frac{13}{5}\Rightarrow\left(5,-\frac{13}{5}\right)$ B

20) Integrating the derivative of *f*(x) is simply just *f*(x).$\Rightarrow$ The area= *f*(8)-*f*(-4)= 432 E

21) $y=Ce^{rt}\Rightarrow y=1.2\times10^6e^0\therefore C=1.2\times10^6\Rightarrow 1.35\times10^6=1.2\times10^6e^{5r}\therefore r=.0235566\Rightarrow$
$y=1.2\times10^6e^{45\times.0235566}=3.46\times10^6$ A

22) $\int_3^{\alpha}\ln(x-2)dx=\int_{\alpha}^{10}\ln(x-2)dx\Rightarrow(\alpha-2)\ln(\alpha-2)+3=-(\alpha-2)\ln(\alpha-2)+\alpha+24\ln 2-10\Rightarrow\alpha=7.466$ C

23) $\int\log x dx\Rightarrow u=\log x=\frac{\ln x}{\ln 10}$, $dv=1dx$, $du=\frac{dx}{x\ln 10}$, $v=x\Rightarrow\int udv=uv-\int vdu=\frac{x\ln x}{\ln 10}-\int\frac{x}{x\ln 10}dx=$
$\frac{x\ln x}{\ln 10}-\frac{x}{\ln 10}+C=\frac{x(\ln x-1)}{\ln 10}+C$ C

24) $(3x^2+9)\frac{dy}{dx}=xy\Rightarrow\frac{dy}{y}=\frac{x}{3x^2+9}dx$, integrating$\Rightarrow\ln y=\frac{\ln(x^2+3)}{6}+C$, exponentiating$\Rightarrow$ $y=C(x^2+3)^{\frac{1}{6}}$ D

25) $\int_0^1(e^{2x}+1)^2dx=\int_0^1(e^{4x}+2e^{2x}+1)dx=\frac{e^{4x}}{4}+e^{2x}+x\Big|_0^1=\frac{e^4}{4}+e^2-\frac{1}{4}$ A

26) Volume= $\int_2^4\frac{\pi}{2}\left(\frac{3x^2-6x}{2}\right)^2dx=\frac{186\pi}{5}$ A

27) $V_x=\pi\int_0^8 x^{\frac{2}{3}}dx=\frac{96\pi}{5}$, $V_y=2\pi\int_0^8 xx^{\frac{1}{3}}dx=\frac{768\pi}{7}$, $V_8=2\pi\int_0^8(8-x)x^{\frac{1}{3}}dx=\frac{576\pi}{7}\Rightarrow\frac{96\pi}{5}<\frac{576\pi}{7}<\frac{768\pi}{7}$
x-axis, $x=8$, y-axis C

28) $\int\frac{x}{\sqrt{x}}dx=\int x^{\frac{1}{2}}dx=\frac{2x^{\frac{3}{2}}}{3}+C$ D

29) Sphere at origin= $y=\sqrt{49-x^2}$, $2=\sqrt{49-x^2}\Rightarrow x=\pm\sqrt{45}$, Volume of ring=
$\pi\int_{-\sqrt{45}}^{\sqrt{45}}\left[\left(\sqrt{49-x^2}\right)^2-2^2\right]=180\pi\sqrt{5}$ B

30) The function $t\sqrt{2t^2+3}$ is continuous over the entire real line, thus the Second Fundamental Theorem of Calculus can be used. $\frac{d}{dx}\left[\int_0^x t\sqrt{2t^2+3}dt\right]=x\sqrt{2x^2+3}$ B