

Solutions Alpha Topic Test – Equations and Inequalities
FAMAT State Convention 2003

For all questions, answer “E) NOTA” means none of the above answers is correct.

1) $D = 5$. $2^{k-5}16^{k+5} = 4^{5k-5}$ reduces to $32768(32^x) = \frac{1024^x}{1024}$. Use natural log to reduce to $5x\ln(2) + 15\ln(2) = 10x\ln(2) - 10\ln(2)$. Solving yields $x = 5$.

2) $D = \frac{41}{21}$. The solutions are $\frac{7}{2}$ and $\frac{3}{5}$. So $\frac{2}{7} + \frac{5}{3} = \frac{41}{21}$.

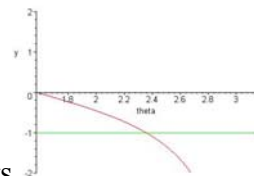
3) $B = 0.12$. Consider the distribution function line from $(7,0)$ on the X-axis and $(0,7)$ on the Y-axis. The area below is satisfies $X + Y \leq 7$. The area below it is $49/2$, or 24.5 . The total area (10×20) is 200 . $49/400 = 0.12$.

4) $A = \frac{-1}{4} < t < \frac{7}{2}$. Separate equations to find $t-5 < -t+2$ while also $-t+2 < 7t+4$. Solve individually.

5) $C = \frac{\pi}{4}$. The lines create a sector centered at the with perpendicular (90°) cross radii. Thus total area divided by 4 yields $\frac{\pi}{4}$.

6) $A = \frac{3\pi}{4} \leq \theta < \pi$. The graph is shown to the right. Using the properties of $\tan(\theta)$, we can translate them to $\cot(\theta)$ and determine when this will drop below -1 .

7) $B = \frac{x-3}{7}$. Rewrite as $x = 7y + 3$ and solve for y .



8) $C = 11$. The inequality reduces to $x < \frac{191}{18} = 10.611$. There are 11 non-negative integers Less than 10.6 $(0,1,2,3 \dots 8,9,10)$.

9) $C = 127$. Use factoring rules to reduce to $(x-7)(x-3)(x+4) = 0$. So $l = -4$, $m = 3$, $n = 7$.

10) $A = \frac{-751}{155}$. This problem uses one recursion step. $f(5)$ means $\frac{f(1/5) - 5^2}{1+5} = f(5)$. Now you have to find

$f(1/5)$ which yields $f(5) - 0.2^2 = (1 + 0.2)f(1/5)$. Combine and substitute these to get $\frac{-751}{155} = f(5)$

11) $B = \frac{-5}{44}$. Plug $g(x) = \frac{x}{x+1}$ back into itself a few times to realize the pattern, you will arrive at $\frac{x}{9x+1} = 5$.

12) $B = 146$. Reduce the first equation to $x = \frac{36\sqrt{14}}{\sqrt{y}}$, the third to $x = \frac{93312}{z^2}$, and the second to $z = \frac{225792}{y^2}$.

Now use the reduced second and substitute into the reduced third equation to get $x = \frac{9y^4}{4917248}$. Now combine

that with the reduced first equation, and the variable remaining is y , found to be 56 . Similarly for the others to find $x = 18$, $z = 72$. $72+18+56 = 146$.

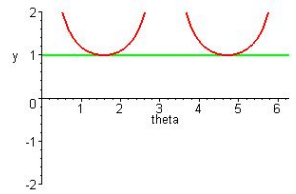
13) $D = 54$. Use log properties to reduce to $\ln(5)\ln(6x^2) + \ln(15)\ln(x) + \ln^2(3) = 3.584756\ln(15)(5)$ to find $x = 7$. $x^2 + 5$ to find 54 .

14) $C = \frac{-33}{19} < x < \frac{-7}{5}$. You have to break this into two cases where $5x + 7$ is positive, and one when negative.

This yields the two cases with two conditions for the answer.

15) $B = -4$. The denominator factors to $(x+5)(x-7)$. This is the base for the decomposition. Setup $\frac{Q}{x+5} + \frac{P}{x-7}$, use common denominators and get a result of $A = 13$, $B = 17$. Thus $A - B = -4$.

16) $D = \frac{\pi}{2}, \frac{3\pi}{2}$. The graph is shown below. $|\csc(\theta)| = 1$ when $\sin(\theta) = 1$. Thus $\frac{\pi}{2}, \frac{3\pi}{2}$.



17) $A = \frac{1}{\arcsin\left(\frac{1}{x}\right)}$. Rewrite as $x = \csc\left(\frac{1}{y}\right)$, then $x = \frac{1}{\sin(1/y)}$, $\sin(1/y) = \frac{1}{x}$

then to $\frac{1}{y} = \arcsin\left(\frac{1}{x}\right)$, then to $\frac{1}{\arcsin\left(\frac{1}{x}\right)} = y$.

18) $C = 2\pi$. The period shifts cancel out, period remains 2π .

19) $A = (0, 1]$. Arcsin(x) can only accept values between 0 and 1. The 0 case is not acceptable for $\ln(0)$. Thus the $(0, 1]$ interval choice.

20) $B = 108$. Factor out a 4 to yield $4(x^2 + 12x + k/4)$. Replace $k/4$ with c . Factor to $(x-c)(x-3c)$. Expand out and compare terms to find $c = 27$, so $k = 108$.

21) $D = 4$. $x^9 + 1 = (x+1)(x^2 - x + 1)(x^6 - x^3 + 1)$. So $A=B=C=D=1$. So $4(1) = 4$.

22) $D = \frac{49}{8}$. Determinant reduces to $2(5x-23)$ which equals $2k + 3$. Solve to find $k = \frac{49}{8}$

23) $A = \frac{-11}{6}$. Again, you must break this into two cases: x positive, and x negative. This eliminates the absolute

value signs and creates two cases. $\frac{8}{3}, \frac{-9}{2}$.

24) $C = 7$. Plug in $(x-1)$ into the equal to find $f(x) = 2x+4$. $18 = 2x + 4$, so $x = 7$.

25) $B = 1$. Substitute $k = \cos(x)$ to yield $4k^2 - 2k - 6 = 0$. Thus $k = 1.5$ or -1 . $\cos(x) = -1$ only once on the interval from $[0, 2\pi]$ and can't reach 1.5, so only once.

26) $A = -5$. Use synthetic division and possible roots to reduce the equation to $(x-3)(x-1)(x+1)(x+3)(x+5)$. So the roots are 3, 1, -1, -3, -5. So the sum = -5.

27) $B = -4 + \sqrt{17} < x$. Again break into two cases of x positive and x negative.

28) $C = 0$. The only way for these to be equal is for both to be 1. Thus $k = 0$ for all cases.

29) $D = 1 - \sqrt{2}$. Reduce the equation to $(|t|+1)(t+1) = 2|t|$. Then further reduce to $-|t| = \frac{t+1}{t-1}$. This clearly leads to

taking two cases for t positive and t negative. Yielding the joint region of $1 - \sqrt{2}$ as a solution.

30) $A = -5$. The equation can be modeled by a continuous quadratic, so the minimum value occurs at

$\frac{-b}{2a} = \frac{-15}{32} = -.46875$. The closest integer is zero, so $p(0) = -5$.