

2003 Theta Circles Topic Test – Worked Out Solutions

1. C $3x - 40 + x = 360 \rightarrow x = 100$; $3x - 40 = 3(100) - 40 = 260$
2. A The circumcenter is the center of the circumscribed circle.
3. A $AB = 2$; $BD = 1$; $DE = 1$; $2 + 1 + 1 = 4$
4. C $AB = 7$; $BC = 4$; $P = 2(7 + 4) = 22$
5. C Let $OC = x$; $\frac{1}{2}x^2 = 12.5 \rightarrow x = 5$; $A = \pi(5)^2 = 25\pi$
6. C $r = \frac{17\sqrt{3}}{3}$; $A = \pi\left(\frac{17\sqrt{3}}{3}\right)^2 = \frac{289\pi}{3}$
7. B $BG = 3.6$; $AG = 6.4$; $BE = 6$; $EG = 4.8$; $DG = 4.8$; Area of $ADBE = \frac{1}{2}(9.6)(3.6) + \frac{1}{2}(9.6)(6.4) = 48$
8. B $18 - x = 4 + x \rightarrow x = 7$; $3 + 7 + 11 = 21$
9. B $12 \bullet 12 = (2r - 4)(4) \rightarrow r = 20$
10. D $1 : 35 + 0 : 20 = 1 : 55$; $30 + 30 + 27.5 = 87.5$
11. E $A = \frac{1}{2} \bullet \pi \bullet 5^2 - \frac{1}{2}(10 \sin 20^\circ)(10 \cos 20^\circ) \approx 23.2$
12. D
$$\begin{cases} \frac{(3x + y) + (2x + y)}{2} = x + 2y \rightarrow xy = 20 \bullet 30 = 600 \\ (x + 2y) + (2x + 2y) = 180 \end{cases}$$
13. D $3x \bullet x = 8 \bullet 8 \rightarrow x = \frac{8\sqrt{3}}{3} \rightarrow r = \frac{16\sqrt{3}}{3}$
14. C $\pi(18\sqrt{2})^2 - \frac{1}{2}(18\sqrt{3})(27) = 648\pi - 243\sqrt{3}$
15. A $9^2 + (AQ)^2 = 15^2 \rightarrow AQ = 12$; $6^2 + (PB)^2 = 15^2 \rightarrow PB = 3\sqrt{21}$; $PB + QA = 12 + 3\sqrt{21}$
16. B $\frac{18 - x}{x} = \frac{6}{3} \rightarrow x = 6$; $CD = 6\sqrt{3} + 3\sqrt{3} = 9\sqrt{3}$
17. A $A = \frac{1}{2}(6\sqrt{3} + 6\sqrt{5})3$

18. B $5^2 + (CD)^2 = 13^2 \rightarrow CD = 12$

19. A $\frac{4}{8} = \frac{3}{QR} \rightarrow QR = 6$

20. C $\frac{(4x-4y)-(x-y)}{2} = 30 \rightarrow x+y = 60+40 = 100$

21. D $6 \bullet 6 = AD \bullet 4 \rightarrow AD = 9 \rightarrow d = 9+4 = 13$

22. C $r^2 + 4^2 = 5^2 \rightarrow r = 3; A = \pi(3)^2 = 9\pi$

23. C $r = 2 \rightarrow V = \frac{2}{3}\pi(2)^3 = \frac{16\pi}{3}$

24. D Perpendicular Bisector of \overline{AB} : $y = -\frac{2}{3}x + 3$; Perpendicular Bisector of \overline{BC} : $y = -5x + 42$;
 \overline{AB} and \overline{BC} intersect at (9,-3), which is the center; $r = \sqrt{65}$;
 $(x-9)^2 + (y+3)^2 = 65 \rightarrow x^2 + y^2 - 18x + 6y + 25 = 0$

25. C The tangent line is perpendicular to the segment whose endpoints are the centers of the circle; $y-4 = \frac{4}{3}(x-9) \rightarrow y = \frac{4}{3}x - 8$

26. B $\cos 40 = \frac{x}{4000} \rightarrow x \approx 3064.18$

27. B $\frac{6\pi \text{ in.}}{\text{sec.}} = \frac{12\pi \text{ in.}}{x} \rightarrow x = 2$

28. D $4^2 + (-5)^2 > 36$, so (4,-5) lies in the exterior.

29. A $5^2 + (BE)^2 = 20^2 \rightarrow BE = 5\sqrt{15}$

30. E Statements A, B, C, and D are true.