# Team Solutions—Calculus Bowl 2003
# FAMAT State Convention

1. Cannot be determined. Since $\ln w^3$ is not everywhere defined on any open interval containing -3, the 2nd Fund. Thm of Calculus does not apply.

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2. 4 All are true except (iv). X = 0 is a removable discontinuity, not an aysmptote.

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3. $(-\pi,\pi)$ This is an even function and the answer is found by inspection and knowing the cosine curve.

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4. 10.402 For A: $\lim_{x\to\pi}\frac{f(x)}{g(x)}=\lim_{x\to\pi}\frac{\frac{3x^2}{2}}{1-\cos x}=\frac{3\pi^2}{4}\approx 7.40220$ For B:

$\lim_{h\to 0}\frac{\int_1^{1+h}\sqrt{x^3+8}\,dx}{h}=\lim_{h\to 0}\sqrt{(1+h)^3+8}=3$ Therefore, A + B (to the nearest thousandth) = 10.402

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5. 0.594 $f'(x)=e^x\cos e^x \Rightarrow e^c\cos e^c=\sin e-\sin 1\Rightarrow c=0.594$

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6. 11.982 Find A - B by $\left|V_x-V_y\right|=\left|\pi\int_0^{\sqrt{3}}(-x^2+3)^2dx-\pi\int_0^3(3-y)dy\right|$

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7. $\frac{\pi}{2}$ Using Euler's Method: $y_0=(0,1)\Rightarrow y_0'=m=1\Rightarrow \Delta y=m\Delta x=\frac{\pi}{2}$, then

$y_1=(\frac{\pi}{2},1+\frac{\pi}{2})\Rightarrow y_1'=m=0\Rightarrow \Delta y=m\Delta x=0$ and $y_2=(\pi,1+\frac{\pi}{2})\Rightarrow y(\pi)=1+\frac{\pi}{2}$

Using derivatives: $\frac{dy}{y}=\cos x\,dx\Rightarrow \ln y=\sin x+c\Rightarrow y=e^{\sin x}\Rightarrow y(\pi)=1$

The error: $\varepsilon=\left|1+\frac{\pi}{2}-1\right|=\frac{\pi}{2}$

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8. 19.590 The rate of growth goes thru 3 distinct regions: I-introduction of frogs(slow growth), II-Rapid growth of frogs, III-Declining growth. A relative maximum will occur in region II. At this

point in time the 2nd derivative =0 (point of inflection) and
$P'(t) = -450(1+950e^{-.35t})^{-2}(950e^{-.35t})(-.35) \Rightarrow$ maximum at $t = 19.590$

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9. $\boxed{101.517^\circ}$

$$T'(t) = \frac{dT}{dt} - 0.3(T-72) \Rightarrow \frac{dT}{T-72} = -0.3dt \Rightarrow \ln|T-72| = -0.3t + C_1 \Rightarrow T(t) = C_2e^{-0.3t} + 72$$

$$T(0) = C_2 + 72 = 170 \Rightarrow C_2 = 98 \Rightarrow T(5) = 98e^{-1.2} + 72 \approx 101.517$$

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10. $\boxed{14}$ Plugging x = ½ into the equation, we get y = 3 or 4 and $\frac{dy}{dx} = \frac{7y - 112x}{y - 7x}$. Plugging in ( ½ , 3) and ( ½ , 4) gives the slopes of 70 and −56, and their sum is 14.

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11. $\boxed{-2}$ $\int_{x_1}^{C_1}[f(x) - g(x)] = F(C_1) - F(x_1) = \frac{A}{3}$ and $\int_{C_2}^{x_2}[f(x) - g(x)] = F(x_2) - F(C_2) = \frac{A}{3}$.
Plugging in the intersection points (-2.732 and 0.732) for $x_1$ and $x_2$ respectively, and plugging in the area between the curves for A (6.928), will give $C_1 = -1.392$ and $C_2 = -0.608$ and their sum is −2.

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12. $\boxed{18\ ¼}$ To find A-- $k(x) = g(f(x)) \Rightarrow k'(x) = g'(f(x))f'(x) \Rightarrow k'(1) = g'(2)(4) = 2 \bullet 4 = 8$.

To find B-- $k(x) = g(x^2) \Rightarrow k'(x) = g'(x^2)2x \Rightarrow k'(1) = g'(1)(2) = 5 \bullet 2 = 10$.

To find C--

$$k = f^{-1} \Rightarrow k'(x) = \frac{1}{f'(k(x))} \Rightarrow k'(2) = \frac{1}{f'(k(2))} = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(1)} = \frac{1}{4}.$$

So, A + B + C = 8 + 10 + ¼ = 18 ¼ .

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13. $\boxed{\$70{,}286}$ $A_1 = P(1+\frac{r}{n})^{nt} = 20\left(1+\frac{.08}{12}\right)^{12 \bullet 40} = 20(1.00\overline{6})^{480}$ and
$A_2 = 20(1.00\overline{6})^{479} \ldots A_{480} = 20(1.00\overline{6})$.

$$\sum_{n=1}^{480} 20(1.00\overline{6})\sum_{n=0}^{479}(1.00\overline{6})^n = \frac{20.1\overline{3}(1-1.00\overline{6}^{480})}{1-1.00\overline{6}} = \$70{,}286$$

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14. $\boxed{12.190}$ $\frac{dv}{dt} = \frac{k}{v} \Rightarrow vdv = kdt \Rightarrow \frac{v^2}{2} = kt + c \Rightarrow v = \sqrt{2kt+c}$ and $v(0) = \sqrt{c} = 8$ and $v(1) = 6$.

So, $v = \sqrt{64-28t}$ and the domain is $0 \le t \le \frac{16}{7}$. The velocity function has no other zeros in the *domain interval, so total distance is given by*

$$\int_0^{\frac{16}{7}} v(t)\,dt = 12.190$$