Calculus Individual Test Solutions
FAMAT State Convention 2003

1. B-- $F'(x) = \frac{-4}{x^2}$ and is always negative. Hence, B is true. Note: D is not true b/c it includes 0.

---

2. A—Let $y = \sqrt{x} \Rightarrow dy = \frac{1}{2\sqrt{x}} dx$. Plugging in x = 4 and dx = -0.7, gives dy = -0.175. Adding that to $\sqrt{4}$ gives the answer.

---

3. B-- $f''(x) = x^2, f'(0) = 7 \Rightarrow f'(x) = \frac{x^3}{3} + c_1$. Plugging in (0, 7), gives $c_1 = 7$. Therefore, $f(x) = \frac{x^4}{12} + 7x + c_2$ and $c_2 = 2$, which gives the answer.

---

4. D—Checking the answers, you find that $\int_0^3 f(x)dx = \int_0^9 f(x)dx - \int_3^9 f(x)dx = 5 - (-1) = 6$.

---

5. B—Let $u = x+1$, then $du = dx$ and $u - 1 = x$.

$\int (u-1)u^{1/2}du = \int u^{3/2}du - \int u^{1/2}du = \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + c$.

Subsituting back in for x and simplifying gives the answer.

---

6. B--

$\lim_{x\to\infty} \frac{3}{x^2}(2+4+6+\ldots+2n) = \lim_{x\to\infty} \frac{3\cdot 2}{x^2}(1+2+3+\ldots+n)$

$= \lim_{x\to\infty} \frac{6x(x+1)}{2x^2} = \lim_{x\to\infty} 3 + \frac{3}{x} = 3$

---

7. C--

$A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$ & $\frac{dA}{dt} = 2\frac{dr}{dt}$ (given)

$\frac{dA}{dt} = \frac{dA}{dr}\frac{dr}{dt} = 2\pi r\frac{dr}{dt} \Rightarrow 2\frac{dr}{dt} = 2\pi r\frac{dr}{dt} \Rightarrow \pi r = 1$ and $r = \frac{1}{\pi}$.

---

8. C—Consider $f(x) - g(x) = Q(x)$. Then, $Q'(x) = f'(x) - g'(x) > 0$ for all x. Therefore, Q(x) is a monotonic, increasing function, hence Q(x) could be 0 (f(x) = g(x)) only once at most.

---

9. E-- $y = (x+1)\text{Arc}\tan x$

And $\frac{dy}{dx} = (x+1)\frac{1}{1+x^2} + \text{Arc}\tan x$, and

$\frac{d^2y}{dx^2} = \frac{1+x^2-2x(x+1)}{(1+x^2)^2} + \frac{1}{1+x^2} = \frac{2-2x}{(1+x^2)^2}$,

and $\frac{d^2y}{dx^2}$ = 0 at x = 1 and y = $\frac{\pi}{2}$.

---

10. D—At x = 2, $\frac{dy}{dx} = 0$ and $\frac{d^2y}{dx^2} = 2(2) - 5 = -1$. This is negative, therefore a maximum.

---

11. B-- $\frac{dV}{dt} = 12, \frac{dh}{dt} = 4\frac{dr}{dt}$. Using $V = \frac{1}{3}\pi r^2 h$,

$\frac{dV}{dt} = \frac{1}{3}(\pi r^2 \frac{dh}{dt} + h(2\pi r)\frac{dr}{dt})$. Plugging in the given info gives $\frac{dr}{dt} = \frac{3}{4\pi}$.

Using $SA = \pi r^2$, then $\frac{dSA}{dt} = 2\pi r\frac{dr}{dt}$. Plugging in gives the answer.

---

12. B-- $y = x^3 + k \Rightarrow y' = 3x^2$.

3x – 4y = 0 $\Rightarrow y = \frac{3x}{4}$. So,

$3x^2 = \frac{3}{4} \Rightarrow x = \pm\frac{1}{2}$. Using x = ½ (1st quadrant) gives y = 3/8. Plugging in, gives k = ¼ .

13. D-- $\frac{dy}{dx}=\frac{1}{x^2+y^2}\bullet(2x+2y\frac{dy}{dx})@(1,0)=2$

14. B—Area = A(trapezoid)- A(under curve)=

½ (45) - $\int_{-1}^{8}x^{2/3}dx=\frac{27}{10}$

15. B—Equation of the ellipse is $\frac{x^2}{9}+\frac{4y^2}{9}=1$ and $4y^2=9-x^2$. The length of a side is 2y, so

$V=2\int_0^3(9-x^2)dx=36$.

16. C—

$\int_0^1 x^{-2}dx=\lim_{a\to0}\int_a^1 x^{-2}dx=\lim_{a\to0}\left(\frac{-1}{a}+1\right)$ (div.)

$\int_0^2\frac{\ln x}{x}dx=\lim_{a\to0}\int_a^2\frac{\ln x}{x}dx=\lim_{a\to0}\frac{\ln^2 2-\ln^2 a}{2}$ (div.)

$\int_1^\infty e^{-x}dx=\lim_{a\to\infty}\int_1^a e^{-x}dx=\lim_{a\to\infty}\left(e^{-1}-e^{-a}\right)=\frac{1}{e}$ (conv.)

17. C--

$V=\pi r^2h\Rightarrow\frac{dV}{dt}=6=36\pi\frac{dh}{dt}\Rightarrow\frac{dh}{dt}=\frac{1}{6\pi}$

18. C-- $f'(4)=\int_2^4 f''(x)dx=f'(4)-f'(2)=2$.

So, $f'(4)=2+f'(2)=5$

19. B—Using integration by parts, and let $u=x$ and $dv=e^{-2x}$. So,

$\int xe^{-2x}dx=\frac{-1}{2}xe^{-2x}+\frac{1}{2}\int e^{-2x}dx=\frac{1}{4}+\lim_{a\to\infty}\frac{-2a-1}{4e^{2a}}$

$=\frac{1}{4}+0=\frac{1}{4}$.

20. A-- $dy=dx\Rightarrow\frac{dy}{dx}=1=2x\Rightarrow x=\frac{1}{2}$.

21. B--

$f_0=(1,2)\Rightarrow f_0'=m=0\Rightarrow\Delta y=m\,\Delta x=0.$

$f_1=(0,2+0)\Rightarrow f_1'=m=0\Rightarrow\Delta y=m\,\Delta x=0.$

$f_2=(-1,2+0)\Rightarrow f_2'=m=2\Rightarrow\Delta y=m\,\Delta x=-2.$

$f_3=(-2,2-2)\Rightarrow f(-2)=0.$

22. B-- $\int_0^1\frac{40}{1+t^2}dt=40\tan^{-1}t\Big|_0^1=10\pi$

23. D-- $\frac{dP}{dt}=2\pi\frac{1}{2}\left(\frac{L}{9.81}\right)^{-\frac{1}{2}}\frac{1}{9.81}\frac{dL}{dt}$ =-0.058

24. C-- $h'(x)=\frac{1}{3}(x-k)^{\frac{-2}{3}}\frac{2}{3}(2x-k)^{-\frac{1}{3}}$.

Hence,

h' is undefined at the given point (cusp).

25. B—I. False $\Rightarrow\lim_{x\to\infty}\frac{3}{1+e^{\frac{1}{x}}}=\frac{3}{2}$.

II. True $\Rightarrow$ The left and right hand limits as $x\to0$ are not equal.

III. False $\Rightarrow f$ achieves the value of 3 only in its limit.

26. A—Since $\sin r$ is an odd function, F(1) = 0.

27. D--

$\frac{dx}{dt}=2\cos t,\ \frac{dy}{dt}=-2\sin 2t,\ \frac{dy}{dx}=-2\sin t$

$\frac{d^2y}{dx^2}=\frac{-2\cos t}{2\cos t}=-1.$

28. A—Let u = x + c and du = dx. So,

$$\int_{0}^{1+c} g(u)du = 10 = -\int_{1+c}^{0} g(u)du \Rightarrow \int_{1+c}^{0} g(x)dx = -10$$

---

29. B--

$$d^2 = 2s^2 \Rightarrow d = \sqrt{2}s \Rightarrow d'(t) = \sqrt{2}s'(t) = 2t$$

$$\Rightarrow s'(t) = \sqrt{2}t .$$

---

30. C-- $\overline{A} = \frac{1}{4.852}\int_{0}^{4.852} 4e^{\frac{-t}{7}}dt = 2.885 .$
