2003 Calculus Applications

(1) all are antiderivatives, but none include "+ C" term

$\therefore$ $\boxed{E}$

(2) $F = m' \cdot v + m \cdot v'$

$v(10) = 10\frac{m}{s}$ $m(10) = 10\,kg$

$v' = 1\frac{m}{s^2}$ $m' = 1\frac{kg}{s}$

$F = (1\frac{kg}{s})(10\frac{m}{s}) + (10\,kg)(1\frac{m}{s^2}) = 20N$

$\boxed{B}$

(3) $Vol = \pi\int_{-6}^{6}\left(-\frac{x^2}{12}+3\right)^2 dx = 2\pi\int_{0}^{6}\left[\frac{x^4}{144} - \frac{x^2}{2} + 9\right]dx$

$= \frac{288}{5}\pi$ $\boxed{A}$

(4) derivative of a constant $= 0$. $\boxed{C}$

(5) $\left(\frac{\$1}{burger}\right)\left(\frac{burger}{2\times10^6 J}\right)\left(\frac{1055\times10^6 J}{MBtu}\right) = \frac{\$527.50}{MBtu}$

$\boxed{D}$

(6) $\frac{n(n+1)}{2}$ circles (i.e. triangular #)

$area = \pi r^2\left(\frac{n^2+n}{2}\right)$

$\lim_{n\to\infty}\left[\pi\frac{s^2}{4(n+\sqrt{3}-1)^2}\cdot\frac{n^2+n}{2}\right] = \lim_{n\to\infty}\left[\frac{\pi s^2}{8}\cdot\frac{n^2+n}{(n+\sqrt{3}-1)^2}\right]$

$= \frac{\pi s^2}{8}$ $\boxed{A}$

(7) $\frac{ds}{dt}$ = velocity

$\frac{d^2s}{dt^2}$ = acceleration

$\frac{d^3s}{dt^3}$ = jerk $\boxed{C}$

(8) $T - T_s = (T_0 - T_s)e^{-kt}$

10 min: $0 - 80 = (-60-80)e^{-k\cdot10} \Rightarrow k \approx 0.05596$

X min: $32 - 80 = (-60-80)e^{-0.05596\cdot X}$

$X \approx 19$ min $\boxed{D}$

(9) $\rho'(x) = \frac{1}{\sqrt{2\pi}}\cdot(-x)e^{-\frac{x^2}{2}}$

$\rho''(x) = \frac{1}{\sqrt{2\pi}}\left(-e^{-\frac{x^2}{2}} + x^2 e^{-\frac{x^2}{2}}\right) = 0$

$-1 + x^2 = 0 \Rightarrow |x| = 1$ $\boxed{B}$

(10) $F = kx$ (Hooke's Law)

$dW = F\,dx \Rightarrow W = \int_0^{0.5} 10x\,dx = \frac{5}{4}J$ $\boxed{E}$

(11) $X_2 = X_1 - \frac{f(x)}{f'(x)} = 2.5 - \frac{2.5^2 - 5}{2(2.5)}$

$X_2 = 2.25$ $\boxed{E}$

(12) $\frac{dV}{dt} = 1\frac{m^3}{s} = 4\pi r^2\cdot\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{4}{\pi}\frac{m}{s}$

$\frac{dA}{dt} = 8\pi r\cdot\frac{dr}{dt} = 8\frac{m^2}{s}$ $\boxed{C}$

(13) i) $y'' = -\cos(x) - \sin(x)$ ✓

ii) $y'' = y$ X

iii) $y'' = -\cos(x) + e^{-x}$ X

iv) $y'' = y = 0$ ✓

$\boxed{D}$

(14) at $x = 5$, $\frac{dM}{dx} = 400$ $\boxed{B}$

(15) $f'(x) = \cos(x) + 1 \geq 0$

$\therefore$ no local maxima exist

$\boxed{A}$

(16) $a(t) = x''(t) = -\omega^2 A\cos(\omega t) = -\omega^2\cdot x(t)$

$\therefore K = -\omega^2$ $\boxed{E}$

(17) $\lim_{t\to\infty} Ae^{-ct}\cos(\omega t) = 0$ $\boxed{A}$

(e^{-ct} term dominates)

(18) $\lim_{x\to\infty}\left[\sqrt{3x^2+2x+1}-\sqrt{3x^2-x}\right]$

$x=\frac{1}{y} \Rightarrow \lim_{y\to 0}\left[\sqrt{\frac{3+2y+y^2}{y^2}}-\sqrt{\frac{3-y}{y^2}}\right]$

$= \lim_{y\to 0}\left[\frac{\sqrt{3+2y+y^2}-\sqrt{3-y}}{y}\right]$

(l'Hopital)

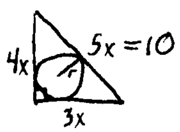
$= \lim_{y\to 0}\left[\frac{\frac{1}{2}(3+2y+y^2)^{-1/2}(2y+2)+(3-y)^{-1/2}\cdot\frac{1}{2}}{1}\right] = \underline{\underline{\frac{\sqrt{3}}{2}}}$ $\boxed{C}$

(19) $\frac{1}{12}\int_0^{12} 80+10\sin\left(\frac{\pi}{12}t\right)dt = 80+\frac{10}{12}\int_0^{12}\sin\left(\frac{\pi}{12}t\right)dt$

$= 80-\frac{5}{6}\cdot\frac{12}{\pi}\left[\cos\left(\frac{\pi t}{12}\right)\right]_0^{12} = 80-\frac{10}{\pi}(-2) = \underline{\underline{80+\frac{20}{\pi}}}$

$\boxed{B}$

(20)



$r = \frac{\text{area of }\Delta}{\text{semi perimeter}} = \frac{6x^2}{6x} = x \text{ in.}$

$x=2$

$\frac{dx}{dt}=\frac{dr}{dt}=1\frac{\text{in.}}{s}$

$A=\pi r^2 \Rightarrow \frac{dA}{dt}=2\pi r\frac{dr}{dt}=2\pi(2)(1\frac{in}{s}) = \underline{\underline{4\pi\ \frac{in^2}{s}}}$ $\boxed{D}$

(21) $f(x)=\sin(x)+x$ $\quad m=1$

$\frac{df(x)}{dx}=\cos(x)+1=1 \quad x=\frac{\pi}{2},\frac{3\pi}{2}$ $\quad\therefore$ $\underline{\underline{\text{2 points}}}$ $\boxed{C}$

(22) $v(t)=at+v_0=at+60=90\frac{mi}{hr} \Rightarrow t=\frac{30}{a}$

$s(t)=\frac{1}{2}at^2+v_0t=\frac{1}{2}a\left(\frac{30}{a}\right)^2+60\left(\frac{30}{a}\right)=1\,mi$

$\underline{\underline{a=2250\ \frac{mi}{hr^2}}}$ $\boxed{D}$

(23) $\frac{dv}{dr}=\frac{-2Pr}{4\eta l}=-\frac{(0.04)(0.002)}{2(2.7\times10^{-7})(2)} \approx \underline{\underline{-74\,\frac{cm/s}{cm}}}$

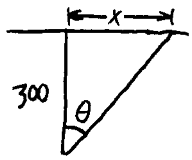
(24) $L(F(t))=\int_0^{t_0}0\,dt+\int_{t_0}^{\infty}F_0e^{-st}dt = F_0\left[-\frac{1}{s}e^{-st}\right]_{t_0}^{\infty}$

$= -\frac{F_0}{s}\left(0-e^{-st_0}\right) = \underline{\underline{\left(\frac{F_0}{s}\right)e^{-st_0}}}$ $\boxed{B}$

(25) $\left.\begin{array}{l} y(0)=0 \Rightarrow A=0 \\ y'(0)=0 \Rightarrow B=0 \\ y''(0)=0 \Rightarrow C=0 \end{array}\right\} \Rightarrow y''(1) = \underline{\underline{D+E+F=1}}$

$\boxed{E}$

(26) $\omega=\frac{2\pi}{20}=\frac{\pi}{10}\frac{rad}{s}=\frac{d\theta}{dt}$



$\tan\theta=\frac{x}{300}$

$300\sec^2\theta\cdot\frac{d\theta}{dt}=\frac{dx}{dt}$

$\frac{dx}{dt}=300\sec^2 45°\cdot\frac{\pi}{10}=\underline{\underline{60\pi\ \frac{ft}{s}}}$

$\boxed{A}$

(27) vertex form: $4p(y-k)=(x-h)^2$ $\quad p=1\ ;\ (h,k)=(0,-1)$

$4(y+1)=x^2 \Rightarrow \underline{\underline{y=0.25x^2-1}}$ $\boxed{B}$

(28) $\#=1000(20+3x)(20-x)=1000(400+40x-3x^2)$

$\frac{d\#}{dx}=1000(40-6x)=0 \Rightarrow x=6\frac{2}{3}$

avg ticket price $= 20-6\frac{2}{3} \approx \underline{\underline{\$13}}$ $\boxed{C}$

(29) $\int dQ(t)\,dt=\int_0^{5RC}I(t)dt \Rightarrow Q(t)=\frac{V}{R}\int_0^{5RC}e^{-\frac{t}{RC}}dt$

$= -VC\left[e^{-\frac{t}{RC}}\right]_0^{5RC} = -VC\left(e^{-5}-1\right) = \underline{\underline{CV\left(1-e^{-5}\right)}}$

$\boxed{A}$

(30) $R(\theta)=\frac{V_0^2}{g}\sin(2\theta)$ $\quad \frac{dR(\theta)}{d\theta}=0 \Rightarrow \theta=45°$

$R(45°)=\frac{190}{10}\cdot(1)=\underline{\underline{19\text{ yard}}}$ $\boxed{D}$