Solutions to Theta Topic Test - Circumference, Perimeter, Area, and Volume
FAMAT State Convention 2003

1) B = 25. Area = (0.5)(base)(height) = (0.5)(10)(5) = 25 = B.

2) D = x^2. From the side x, we can find the side length of the y-square to be y = $\sqrt{2}x$. So the shaded regions area is $\left(\sqrt{2}x\right)^2 - x^2 = x^2$.

3) A = $\frac{4\sqrt{5}}{27}$. The smallest triangle still similar means 1 ~ 9. So the new triangle sides are $1, \frac{8}{9}, \frac{7}{9}$. Thus, using Heron's formula, $s_p = \frac{4}{3}$, so A = $\sqrt{\frac{4}{3}\left(\frac{1}{3}\right)\left(\frac{4}{9}\right)\left(\frac{5}{9}\right)} = \frac{4\sqrt{5}}{27}$.

4) D = $\frac{4}{7}$. The diagonal of $7\sqrt{2}$ means side lengths of 7. Thus the perimeter is 28 with area of 49.

5) C = $4\sqrt{2}$. The area of the triangle implies a diameter of 8. D = 8 means the sides must be equal to length $\frac{8}{\sqrt{2}}$ or $4\sqrt{2}$

6) C = 16. 7x + x = 8x = 180. Thus each interior angle is 157.5. $157.5 = \frac{(n-2)180}{n}$, so n = 16.

7) B = $16\sqrt{3}$. The 2nd term yields side length of 8. Thus A = $\frac{s^2\sqrt{3}}{4} = \frac{8^2\sqrt{3}}{4} = 16\sqrt{3}$.

8) B = 30.5. Setup the matrix $\begin{array}{ccc|c} 5 & 1 & 4 & 5 \\ -7 & 0 & 10 & -7 \end{array}$.

Thus Area = |(0.5) * {(5)(0) + (1)(10) + (4)(-7) — [(-7)(1) + (0)(4) + (10)(5)]}| = |-30.5| = 30.5, B.

9) A = $x - y$ = 8. Plugging in yields endpoints of (22, 14) and (14,16). m =1. y-6=1(x-14).

10) B = 49 π. $(x-4)^2 + (y+7)^2 = 49$. r = 7.

11) D = 29.74 0.65A = C. $0.65\pi r^2 = 2\pi r$. Thus r = 3.0769, so A = 29.74

12) C = 12. (x-1)(x-4)=(x)(x-3). Thus x = 2. 2(6) = 12.

13) D = 40. Each of the 4 triangles has area $\frac{(4)(5)}{2}$. Thus (8)(5) = 40.

14) C = 21.3. The lines intersect at (1.34782, 1.47826). Thus the radius of the circle from that point to (2,4) is r = 2.604. Thus A = $2.604^2\pi = 21.3$.

15) C = 14.8. The size of the arc is 34.6°. $\frac{34.6}{360}\pi(7^2)$ = 14.795 = 14.80.

16) B = 360. The face dimensions factor to (12)(6) = 72, (6)(5) = 30, (12)(5) = 60. Thus the volume = (12)(5)(6) = 360.

17) B = 0.658 = 0.66. Triangle side = 8 , square side = $\sqrt{16\sqrt{3}}$. $\frac{\sqrt{16\sqrt{3}}}{8}$ = 0.658 = 0.66.

18) C = 625. $500 = ($10)(# of feet you can use) … Thus 50 feet available. The optimal split is a square, so 25 feet on each side using the building, yields a square of area 625.

19) B = $\frac{3}{4}x$. Areas of 16:9 means a similarity ratio 4:3. Thus the perimeter of B, is (3:4)x , or $\frac{3}{4}x$.

20) B = $\frac{\pi\sqrt{2}}{3}$. Using 45-45-90 rules, the circumference is $\pi\sqrt{2}$, so radius is $\frac{\sqrt{2}}{2}$. Thus the volume created by rotation is V = $\frac{4}{3}\pi\left(\frac{\sqrt{2}}{2}\right)^3 = \frac{\pi\sqrt{2}}{3}$.

21) A = Board A. Board A = $(5^2 - 4^2) + (3^2 - 2^2) + 1^2 = 15$. Board B = 25 – 16 + 9 – 4 = 14. Thus Board A is bigger.

22) A = $\frac{8}{13}$. $R_{large} = \frac{26}{2\pi}$. $R_{small} = \frac{16}{2\pi}$. $\frac{16/2\pi}{26/2\pi} = \frac{8}{13}$.

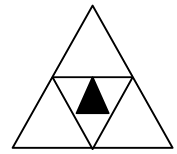
23) D = 120. Area = $\frac{1}{2}(30)(8)=60$.

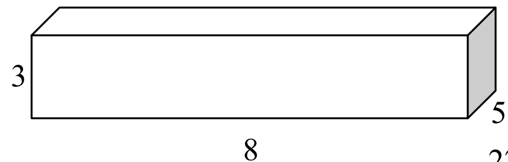
24) C = 25. The area of the circle is (5)(5)π = 25 π. The new circle becomes 625 π. Thus $r_{new} = \sqrt{625} = 25$.

25) D = $\frac{1}{16}$. An approximate sketch is shown. More exactly, the area is (1/4) the original but twice, so $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$.

26) C = $16\pi\sqrt{2}$. LSA = $\pi r(slanthgt) = \pi 4(4\sqrt{2}) = 16\pi\sqrt{2}$.

27) A = 120. The box created is shown at right. V = (8)(5)(3) = 120.





28) D = 69.9 $A_{cir.}=121\pi; r=11$(also triangle altitude). Use 30-60-90 rules to find $s_{tri.} = \frac{22\sqrt{3}}{3}$. $A_{tri} = \frac{\left(\frac{22\sqrt{3}}{3}\right)^2 \sqrt{3}}{4} = 69.859$

29) B = 60. LA = (0.5)(slant height)(base perim.) = (5)(24)(0.5) = 60.

30) C = $5\pi\sqrt{2}$. The radius of the outer circle implies a side length of $\frac{10}{\sqrt{2}}$, or smaller

Circle radius of $\frac{5}{\sqrt{2}}$. So the circumference $=(2)\frac{5}{\sqrt{2}}\pi$ or $5\pi\sqrt{2}$.