

Answer Key Alpha Individual MAO State Convention 2003

1. $\tan x = 2 \sin x$

$$\frac{\sin x}{\cos x} = 2 \sin x \Rightarrow \sin x (2 \cos x - 1) = 0$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi \Rightarrow 3\pi \quad \text{ans C}$$

2. $x = -3$, no horizontal asymptote, $y = -x + 3$

ans A

3. Since 5^x and 5^{-x} are reciprocals smallest output comes from $x = 0$.

Therefore numerator = 2 and range $y \geq 0.5$
ans D

4. $(3x+6)(3x+5) = 9x^2 + 33x + 30$ ans D

5. Curves do not intersect ans A

6. $-3c + 0(4) - 1(1) = -3c - 1$ ans A

7. $\log_3(x-3) = \log_9(5-x)$

$$\log_3(x-3) = .5 \log_3(5-x)$$

$$x-3 = \sqrt{5-x}$$

$$x^2 - 6x + 9 = 5 - x$$

$x = 1$ and 4 but 1 extraneous

ans B

8. Identify random point on either plane $(0, -5, 0)$. Use point-plane distance formula.

$$\frac{3(0) - 1(-5) + 4(0) + 1}{\sqrt{3^2 + (-1)^2 + 4^2}} = \frac{6}{\sqrt{26}} \quad \text{ans C}$$

9. Geometric series = $\frac{\sin^2 x}{1 - \sin^2 x} = 1$

$$\tan^2 x = 1 \quad \tan x = \pm 1 \quad x = \frac{\pi}{4} \text{ and } \frac{3\pi}{4} \quad \text{ans D}$$

10. Sixth term has $x = 2$

$$4^3 n^4 = 64 n^4 \quad \text{ans A}$$

11. $f(2) = 5, f(3) = f(2) + 3 + 1 = 9,$

$$f(4) = f(3) + 4 + 1 = 14, \dots, f(10) = f(9) + 10 + 1 = 65$$

ans C

$$12. \frac{0.2}{0.3} = \frac{2}{3} \quad \text{ans E}$$

$$13. \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3} = x + 4y$$

$$\tan^{-1}(1) = \frac{\pi}{4} = 2x - y$$

$$\frac{7\pi}{12} = 3x + 3y$$

$$x = \frac{7\pi}{36}$$

ans D

14. Sum of roots = 10; if $3+i$ root so is $3-i$. Their sum = 6. Remaining roots are real (5 and -1). ans A

$$\begin{bmatrix} i & j & k \end{bmatrix}$$

$$15. \begin{vmatrix} 5 & 2 & 3 \\ -2 & 5 & 0 \end{vmatrix} = -15i - 6j + 29k \quad \text{ans D}$$

16. $e^x - 20e^{-x} = 8$

$$e^{2x} - 20 = 8e^x$$

$$(e^x - 10)(e^x + 2) = 0$$

$x = \ln 10$ $x = \ln(-2)$ is BAD ans B

17. $x^3 - 27 = (x-3)(x^2 + 3x + 9)$

cancel $x-3$ and substitute $x=3$ ans A

18. $10000 = 2000\left(1 + \frac{.055}{4}\right)^{4n}$

$$n = \frac{\ln 5}{4 \ln\left(1 + \frac{.055}{4}\right)} = 29.46 \text{ yrs} \quad \text{ans C}$$

19. $a_{100} = S_{100} - S_{99}$

$$[3(100)^2 - 5] - [3(99)^2 - 5] = 597 \quad \text{ans A}$$

20. Let d =common difference. Three arithmetic means are $x+d$, $x+2d$, $x+3d$.

This means that $x+4d=y$ and $d=\frac{y-x}{4}$.

$$\text{Average} = x+2d = x+2\left(\frac{y-x}{4}\right) = \frac{x+y}{2} \quad \text{ans D}$$

21. Complete square $2(x-1)^2+6(y+2)^2=13$

Divide by 13. Radii are $\sqrt{\frac{13}{2}}$, $\sqrt{\frac{13}{6}}$.

$$\text{Area} = r_1 r_2 \pi = \frac{13\sqrt{3}}{6} \pi \quad \text{ans C}$$

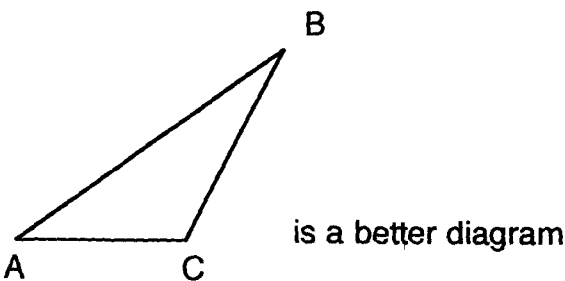
22. Since $c=0$ ans E

$$23. (3\sqrt{2}, \frac{\pi}{4}) = (3, 3)$$

$$(-2\sqrt{2}, -\frac{\pi}{4}) = (-2, 2)$$

$$\text{Distance} = \sqrt{26} \quad \text{ans B}$$

24.



Because of 30-60-90 relationship and triangle formed with altitude from B to AC extension, $AC = 5\sqrt{3} - \sqrt{11}$.

$$\text{Area} = .5(5\sqrt{3} - \sqrt{11})5 = 13.36 \quad \text{ans D}$$

$$25. x^6 + \frac{1}{x^6} = (x^2 + \frac{1}{x^2})(x^4 - 1 + \frac{1}{x^4})$$

$$(x + \frac{1}{x} = 4)^2 \Rightarrow x^2 + \frac{1}{x^2} = 14$$

$$(x^2 + \frac{1}{x^2} = 14)^2 \Rightarrow x^4 + \frac{1}{x^4} = 194$$

$$x^6 + \frac{1}{x^6} = 14(193) = 2702 \quad \text{ans}$$

$$26. \sqrt{i} = (0+i)^{.5} \\ = [1(\text{cis } \frac{\pi}{2})]^{.5} \\ = 1^{.5}(\text{cis } .5 \cdot \frac{\pi}{2}) = \text{cis } \frac{\pi}{4} \quad \text{ans D}$$

$$27. \log(ab) + \log\left(\frac{1}{ab}\right) = \log a + \log b + \log 1 \\ -[\log a + \log b] = 0 \Rightarrow 0 = \log_5 x \Rightarrow x = 5^0 = 1 \quad \text{ans B}$$

$$28. \sqrt[3]{1.2} = (1+.2)^{1/3} \text{ (that's an exponent...)} = \\ 1^{1/3} + \frac{1}{3}(.2) + \frac{\frac{1}{3} \cdot -.2}{2 \cdot 1} (.2^2) = 1.0622 \quad \text{ans A}$$

$$29. (n)^2 \pi - (n-1)^2 \pi = (2n - 1)\pi \quad \text{ans C}$$

30. Parabola is described. Use distance formula \Rightarrow Points on $x=-3$ are $(-3, y)$, on curve (x, y) and then there is $(3, 2)$.

$$x+3 = \sqrt{(x-3)^2 + (y-2)^2}$$

$$x^2+6x+9 = x^2-6x+9+y^2-4y+4$$

$$12x = (y-2)^2. \quad \text{ans B}$$