Alpha Team Key MAO State Convention 2003

1. Given f(x-2) let $x=x+2 \Rightarrow f(x)=(x+2)^2-4(x+2)-5=(x-3)(x+3)$

   $g(\frac{1}{3x})$ let $x=\frac{1}{3x} \Rightarrow g(x)=9x^2$

   $f[g(x)]=(9x^2-3)(9x^2+3)=9(3x^2-1)(3x^2+1)$

   Ans: $9(3x^2-1)(3x^2+1)$

2. From law of sines, $\frac{120}{\sin 47}=\frac{AC}{\sin 72} \Rightarrow AC=156.05$

   let distance across be x, $\sin 61=\frac{x}{156.05} \Rightarrow x=136.48$ **Ans:** 136

3. $a+b=.25$ and $ab=1 \Rightarrow a^2+2ab+b^2=.0625 \Rightarrow a^2+b^2=-1.9375$

   $a^3b+ab^3=ab(a^2+b^2)=1(-1.9375)=-1.9375$ or $\frac{-31}{16}$

   Ans: -1.9375 or $\frac{-31}{16}$

4. Slopes of lines are $\frac{2}{3}$ and $\frac{14}{9} \Rightarrow \tan x=\frac{\frac{2}{3}-\frac{14}{9}}{1+(\frac{2}{3})(\frac{14}{9})} \Rightarrow x=-23.57 \Rightarrow 156.4352938$

   Ans: 156 25'31"

5. $A \Rightarrow (x-2)^2+y^2=\frac{11}{2} \Rightarrow A=\frac{11}{2}\pi$

   $B \Rightarrow (x+3)^2+3(y-1)^2=9 \Rightarrow$ radii are 3 and $\sqrt{3} \Rightarrow B=3\sqrt{3}\pi$

   $C \Rightarrow x^2-4(y-1)^2=4 \Rightarrow$ distance is $\sqrt{1+4}=\sqrt{5}=C$

   $\frac{B}{A}-C=\frac{6\sqrt{3}}{11}-\sqrt{5}$ **Ans:** $\frac{6\sqrt{3}}{11}-\sqrt{5}$

6. Set up synthetic division and work backwards

   2 | **2** **1** **-18** **-9** $\Rightarrow 2x^3+x^2-18x-9 \Rightarrow (2x+1)(x-3)(x+3)$

	4	10	-16
2	5	-8	-25

   Ans: (2x+1)(x-3)(x+3)

7. Use law cosines, $9.1^2=x^2+12^2-24x\cos 35 \Rightarrow 0=x^2-24x\cos 35+61.19$

   Sum of possible x values is $24x\cos 35=19.65 \Rightarrow 20$

   Ans: 20

8. Let $e^x=n \Rightarrow n^2-10n+16=(n-8)(n-2) \Rightarrow e^x=2$ and $8 \Rightarrow x=\ln 2$ and $\ln 8 \Rightarrow \ln 2+\ln 8=\ln 16$

   Ans: 16

9. Because of 30-60-90 and 45-45-90 relations the height is set=x and then the base of 14 becomes $4+x\sqrt{3}+x \Rightarrow x=\frac{10}{1+\sqrt{3}}=5\sqrt{3}-5$.

Ans: $5\sqrt{3}-5$

10. Since $xy^2+x^2y=xy(x+y)$ let $x+y=a$ and $xy=b \Rightarrow ab=30$ and $a+b=11$
$\Rightarrow (a,b)=(5,6)$ and $(6,5)$. Substitute, $x+y=5$ and $xy=6$ as well as $x+y=6$ and $xy=5$.
$\Rightarrow (x,y)=(2,3),(3,2),(1,5),(5,1)$

Ans: (2,3),(3,2),(1,5),(5,1)

11. $\begin{vmatrix} x & 3 & 4 \\ 9 & x & 3 \\ 1 & x & x \end{vmatrix}$ $\begin{matrix} x & 3 \\ 9 & x \\ 1 & x \end{matrix}$ $x=x^3+9+36x-4x-3x^2-27x=24 \Rightarrow x^3-3x^2+5x+9=24 \Rightarrow x^3-3x^2+5x-15$
$\Rightarrow (x^2+5)(x-3)$

Ans: 3

12. $a_1=1$, $a_2=1r$, $a_3=1r^2$, $a_4=1r^3$,...,$a_{10}=1r^9=2 \Rightarrow r=2^{1/9}$.
Multiply $2^{1/9}\cdot 2^{2/9}\cdot 2^{3/9}\cdots 2^{9/9}=2^5=32$. **Ans:** 32

13. $A=[50(51)/2]^2$
$B=50(51)(101)/6$
$C=50(51)/2$ C-E-D+A-B=1,578,925
$D=50(102)/2$
$E=(50)^2$

Ans: 1,578,925

14. $(1+i\sqrt{3})^5=(2\text{cis}\frac{\pi}{3})^5=32\text{cis}\frac{5\pi}{3} \Rightarrow ab=\frac{160\pi}{3}$ **Ans:** $\frac{160\pi}{3}$

15. Consider triangle AGC,
Use law of cosines, $(AC)^2=(AG)^2+(CG)^2-2(AC)(AG)\cos G$
$52=61+9-6(\sqrt{61})\cos G$
$\cos G=\frac{3}{\sqrt{61}} \Rightarrow G=67.4$
$m\angle BFH=90 \Rightarrow \text{sum}=157$

Ans: 157