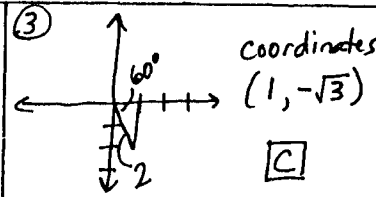


TRIGONOMETRY SOLUTIONS - FAMAT State Convention 2001

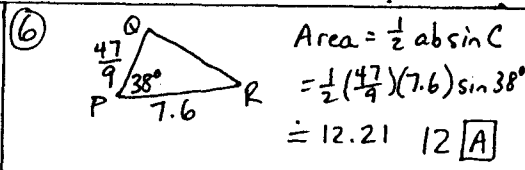
① $\sin \theta + \cos \theta = \frac{4}{5} + \frac{3}{5} = \frac{7}{5}$ **A**

② $\frac{2 \tan \theta}{1 + \tan^2 \theta} = \frac{2 \tan \theta}{\sec^2 \theta} = 2 \frac{\sin \theta}{\cos \theta} \cdot \cos^2 \theta = 2 \sin \theta \cos \theta = \sin 2\theta$ **A**

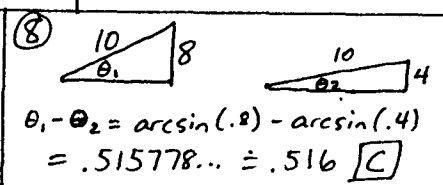


④ $\tan \frac{\pi}{2}, \tan \frac{3\pi}{2}$ undefined
 $(\frac{\pi}{4}, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \frac{3\pi}{4}) \cup (\frac{5\pi}{4}, \frac{3\pi}{2}) \cup (\frac{3\pi}{2}, \frac{7\pi}{4})$ **E**

⑤ $\cos(3x) = 4\cos^3 x - 3\cos x$
 $4 - 3 = 1$ **A**



⑦ II, III, IV, V are odd functions **B**



⑨ $(44 \text{ rad}) (\frac{180 \text{ degrees}}{\pi \text{ rad}}) \approx 2521.01^\circ$
 coterminal with 1.01° @ I **A**

⑩ $\sum_{n=1}^{2001} \sin^2(n\pi) = \sin^2 90 + \sin^2 180 + \sin^2 270 + \sin^2 360 + \dots$
 $= \underbrace{1+0+1+0}_{\text{repeat 500 times}} + 1 = 0+1 = \frac{1}{2}$
 $\sum_{n=1}^{2001} \cos^2(n\pi) = \underbrace{0+1+0+1}_{\text{repeat 500 times}} + 0 = 2(500) + 0 = 1000$ **D**

⑪ $\cos \frac{45}{2} = \sqrt{\frac{1 + \cos 45}{2}} = \sqrt{\frac{1 + \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2 + \sqrt{2}}{4}}$
 $x=2, y=4$
 $10y + x^2 = 40 + 8 = 48$ **D**

⑫ $r = \frac{m}{2} \theta$
 graph of a spiral **E**

⑬ I. per = $\frac{\pi}{8} = \frac{\pi}{4}$
 II. per = $\frac{2\pi}{8} = \frac{\pi}{2}$
 III. per = $\frac{2\pi}{8} = \frac{\pi}{2}$
 I, II, and III **D**

⑭ $(27 \text{ cis } 300)(256 \text{ cis } 320)$
 $(243 \text{ cis } 320)(64 \text{ cis } 300)$
 $= \frac{27 \cdot 256}{243 \cdot 64} = \frac{4}{9}$ **A**

⑮ $g \sec \theta = \frac{\cos^2 \theta + (1 - \sin \theta)^2}{\cos \theta (1 - \sin \theta)} = \frac{\cos^2 \theta + 1 - 2\sin \theta + \sin^2 \theta}{\cos \theta (1 - \sin \theta)}$
 $= \frac{2 - 2\sin \theta}{\cos \theta (1 - \sin \theta)} = \frac{2}{\cos \theta} = 2 \sec \theta$
 $g \sec \theta = 2 \sec \theta$
 $g = 2$ **D**

⑯ $\frac{\sqrt{5}}{4} = \frac{\cot x}{1 - \cot x}$
 $\sqrt{5} - \sqrt{5} \cot x = 4 \cot x$
 $\cot x = \frac{\sqrt{5}}{4 + \sqrt{5}}$
 $\tan x = \frac{4 + \sqrt{5}}{\sqrt{5}} = \frac{4\sqrt{5} + 5}{5}$
 $= \frac{12\sqrt{5} + 15}{15} = \frac{12+5+15}{15} = 32$ **D**

⑰ $7^2 = 4^2 + 6^2 - 2(4)(6) \cos x$
 $49 = 16 + 36 - 48 \cos x$
 $\cos x = \frac{1}{16}$
 $\sec x = 16$ **E**

⑱ $\cot(x-y) = \frac{1 + \tan x \tan y}{\tan x - \tan y}$
 $= \frac{1 + \frac{1}{a} \cdot \frac{1}{b}}{\frac{1}{a} - \frac{1}{b}} = \frac{\frac{ab+1}{ab}}{\frac{b-a}{ab}} = \frac{ab+1}{b-a}$ **E**

⑲ Values of $\sin x$ repeat every 2π (or 360°)
 Prob. = $\frac{(\arcsin .7 - \arcsin .6) \cdot 2}{2\pi}$
 multiply by 2 because there are 2 such intervals
 ≈ 0.042 **D**

⑳ $\sin^2 x + \sin^2 y = 1$
 $\sin^2 x + \cos^2(\frac{\pi}{2} - y) = 1$
 $x = \frac{\pi}{2} - y$
 $x + y = \frac{\pi}{2}$
 $\approx 1.57079...$
 ≈ 1.5708 to 4 places
 $5+7+0+8 = 20$ **C**

㉑ $y = \frac{e^x - e^{-x}}{2} \rightarrow 2y = \frac{e^{2y} - 1}{e^y}$
 $x = \frac{e^y - e^{-y}}{2}$
 $2x = \frac{e^y - e^{-y}}{1}$
 $2x = e^y - \frac{1}{e^y}$
 $e^y = 2x \pm \sqrt{4x^2 + 4}$ **B**

㉒ $\lim_{x \rightarrow 0} \frac{\sin x}{x} + \lim_{x \rightarrow \infty} \frac{\cos x}{x} = 0 + 0 = 0$ **B**

㉓ $x^2 - y^2 = \sec^2 \theta - \tan^2 \theta = 1$
 $x^2 - y^2 = 1$ hyperbola **D**

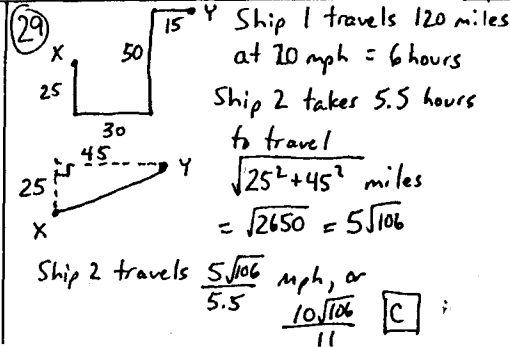
㉔ If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
 $A^{-1} = \frac{1}{-\cos^2 x - \sin^2 x} \begin{bmatrix} \cos x & -\sin x \\ -\sin x & -\cos x \end{bmatrix} = -\begin{bmatrix} \cos x & \sin x \\ \sin x & \cos x \end{bmatrix}$ **B**

㉕ $\frac{x^2}{\frac{1}{4}} + \frac{y^2}{1} = 1$
 Area = $\pi ab = \pi (\frac{1}{2})(1) = \frac{\pi}{2}$
 $\sin(\frac{\pi}{2}) = 1$ **D**

㉖ $\frac{\sec x - 1}{\sec x + 1} = \frac{1}{\cos x} - 1$
 $= \frac{1 - \cos x}{1 + \cos x} = \left(\frac{1 - \cos x}{1 + \cos x} \right)^2$
 $\frac{1}{2}$ angle identity for tan
 $\frac{p}{q} = \frac{1}{2}$
 $p+q = 1+2 = 3$ **A**

㉗ $\csc^2 x (\cos^4 x - \sin^4 x - 1) = \csc^2 x [(\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) - 1] = \csc^2 x (\cos^2 x - \sin^2 x - 1) = \csc^2 x (\cos^2 x - 1 - \sin^2 x) = \csc^2 x (-\sin^2 x - \sin^2 x) = \csc^2 x (-2\sin^2 x) = \frac{-2\sin^2 x}{\sin^2 x} = -2$ **A**

㉘ $\frac{1}{.2} = \frac{2}{.4} = \frac{2}{.6} + \dots$
 $\frac{1}{.4} = \frac{4}{.8} = \frac{4}{.64} + \dots$
 $\frac{1}{.8} = \frac{4}{1.6} = \frac{4}{1.6}$
 $\dots = \arcsin(.4) \approx 56.7$



㉚ $y = \sqrt{\csc x + y}$
 $y^2 = \csc x + y$
 $y^2 - y = \csc x$
 $x = \text{Arccsc}(y^2 - y)$
 $= \text{Arccsc}[(y)(y-1)]$ **A**