Gemini Topic Test Solutions

1) $(1-i)^{2002} = \left((1-i)^2\right)^{1001}$

$(1-i)^2 = -2i \Rightarrow \left((1-i)^2\right)^{1001} = (-2i)^{1001}$

$= -2^{1001} i$

$\Rightarrow D$

2)

$1 - p = 1 - \left(\frac{365}{365}\cdot\frac{364}{365}\cdot\frac{363}{365}\cdots\frac{358}{365}\cdot\frac{357}{365}\right)$

$1 - p \approx 0.91 \Rightarrow p = 0.09 \Rightarrow C$

3) Using the Binomial Theorem,

$100 - \sqrt{9999} = 100 - \sqrt{10{,}000 - 1}$

$= 100 - \left(\sqrt{10{,}000} + \frac{-1}{2\sqrt{10{,}000}}\right)$

$= 100 - \left(100 - \frac{1}{200}\right) = 0.005 \Rightarrow B$

4) $\frac{1}{A} + \frac{1}{J} + \frac{1}{-12} = \frac{1}{x+2}$ & $\frac{1}{A} + \frac{1}{J} = \frac{1}{x}$

$\Rightarrow \frac{1}{-12} = \frac{1}{x+2} - \frac{1}{x} = \frac{-2}{x(x+2)}$

$\Rightarrow x(x+2) = 24 \Rightarrow x = 4 \Rightarrow C$

6) $x = \sqrt{19 + x} \Rightarrow x^2 - x - 19 = 0$

$x = \frac{1 \pm \sqrt{77}}{2} \Rightarrow A = 1,\ B = 77,\ C = 2$

$\frac{B}{2AC} = \frac{77}{4} \Rightarrow A$

7) $A = 8! - (7! \cdot 2!) = 30{,}240$

$B = \frac{(9-1)!}{2} = 20{,}160$

$C = \frac{x(x-3)}{2} = 90 \Rightarrow x = 15$

$\frac{2AC}{B} = 45 \Rightarrow B$

5) Apply the multinomial theorem.

$\Rightarrow C$

9)

$\frac{x+y}{2} = 4 \Rightarrow x + y = 8 \Rightarrow x^2 + 2xy + y^2 = 64$

$\sqrt{xy} = 5 \Rightarrow xy = 25$

Use substitution to see:

$x^2 + 2xy + y^2 = 64 \Rightarrow x^2 + 2(25) + y^2 = 64$

$x^2 + y^2 = 14 \Rightarrow C$

8) If D is changed to "The columns of A form a linearly independent set," then all the statements hold true.

$\Rightarrow D$

10) The log of the number is equal to 2001log 2, which is about 602.36, which has 603 digits. $\Rightarrow D$

11) $9x^2 + 4y^2 + 36x - 24y + 36 = 0$

$\Rightarrow \frac{(x+2)^2}{4} + \frac{(y-3)^2}{9} = 1$

$b = 2,\ a = 3$

$c = \sqrt{a^2 - b^2} = \sqrt{5}$ $\quad \frac{c}{a} = \frac{\sqrt{5}}{3} \Rightarrow A$

12) $\Rightarrow C$

13) $e = \frac{5}{4} \Rightarrow hyperbola \Rightarrow D$

14)

$\frac{11}{\sqrt[3]{4} + \sqrt[3]{6} + \sqrt[3]{9}} \cdot \frac{\sqrt[3]{3} - \sqrt[3]{2}}{\sqrt[3]{3} - \sqrt[3]{2}} = 11\sqrt[3]{3} - 11\sqrt[3]{2}$

$\Rightarrow AB - CD = 33 - 22 = 11 \Rightarrow C$

15) Negative Binomial $\Rightarrow E$

16) Petals $= 4 \times 2 = 8 \Rightarrow B$

17) There exists 16 - 1×1, 9 - 2×2, 4 - 3×3, 1 - 4×4, 9 - $\sqrt{2}\times\sqrt{2}$, 1 - $2\sqrt{2}\times2\sqrt{2}$, 8 - $\sqrt{5}\times\sqrt{5}$, 2 - $\sqrt{10}\times\sqrt{10}$, which sums to 50 paths. $\Rightarrow C$

18) $A - \lambda I = \begin{bmatrix} 2-\lambda & 3 \\ 3 & -6-\lambda \end{bmatrix}$

$\det(A - \lambda I) = \lambda^2 + 4\lambda - 21$

$\lambda = 3, -7 \Rightarrow 2 \Rightarrow B$

19) $\frac{n!}{(n-3)!} = n(n-1)(n-2)$

This implies that numbers must be a product of 3 consecutive numbers.

A: n = 500 $\Rightarrow$ $500\times499\times498$

B: n = 450 $\Rightarrow$ $450\times449\times448$

C: n = 600 $\Rightarrow$ $600\times599\times598$

D: n = 550 $\Rightarrow$ $550\times549\times548$

All are possible values $\Rightarrow E$

20) The lines are parallel and the distance between them is given by $\frac{\left|(C_1 - C_2)\cdot\frac{A_1}{A_2}\right|}{\sqrt{A_1^2 + B_1^2}} = \frac{\left|(8-11)\cdot\frac{5}{5}\right|}{\sqrt{5^2 + 6^2}}$

$\approx 0.38411\ldots \Rightarrow 4 \Rightarrow D$

21) When $y = \log_2 x$, $x > y$, while for $x = \log_5 y$, $y > x$. This implies that the graphs don't intersect at all. Hence, $\Rightarrow A$

22) $\frac{3d}{\frac{d}{350} + \frac{d}{400} + \frac{d}{375}} \approx 373.9 \Rightarrow B$

23) Use Vandermonde's Identity:

$\binom{p+q}{r} = \sum_{k=o}^{r}\binom{p}{r-k}\binom{q}{k} \sum_{k=o}^{2n}\binom{2n}{k}\binom{2n}{k} \Rightarrow p = q = r = 2n$

$\binom{p}{r-k} = \binom{2n}{2n-k} = \binom{2n}{k} \cdot \binom{q}{k} = \binom{2n}{k}$

$\binom{p+q}{r} = \binom{2n+2n}{2n} \Rightarrow D$

24) We get a zero when we multiply a 5 and a 2. So lets count the number of 5's and 2's we have. We get a 5 from each 5, 10, 15, …, 100 which makes twenty 5's. Now, 25, 50, 75, and 100 each have two 5's out of which we have counted one. So, we add 4 more. That gives us a total of twenty-four 5's. $\Rightarrow C$	25) Ball travels down, up, down, up, … $Sum = x+\frac{x}{2}+\frac{x}{2}+\frac{x}{4}+\frac{x}{4}+\ldots$ $= x+2\left(\frac{x}{2}+\frac{x}{4}+\ldots\right)=3x$ Fibonacci product: $1\cdot1\cdot2\cdot3\cdot5\cdot8\cdot13=3120$ $3x=3120 \Rightarrow x=1040 \Rightarrow A$	26) Multiply across and add like terms to get: $0=10(x-5)^2(x-2)(x-1)$ Solutions to this equation: 1, 2, and 5. We cannot make 5 a solution since it is not a solution of the original equation. Hence, product $=2 \Rightarrow B$
27) D is the contrapositive of the original statement. $\Rightarrow D$ 28) While D is the correct answer, it does not ignore the possibility that $x=y$. Since the question specified "some trucks crash into **other** trucks," a more correct answer would be $\exists x \exists y\,(T(x)\wedge T(y)\wedge C(x,y)\wedge x\neq y)$. $\Rightarrow E$	29) Huey beats the fireman at badminton, so Huey is *not* the fireman. Also, we should note that Mr. Dewey lives in Detroit and the brakeman lives exactly half way between Chicago and Detroit. The brakeman's nearest neighbor, who is one of the passengers, earns *exactly* three times as much as the brakeman, but Mr. Louie earns exactly $20K per year, and 20 is not divisible by 3, so Mr. Louie is not the brakeman's nearest neighbor. Also, the brakeman's nearest neighbor must not live in Chicago or Detroit, as the brakeman is equidistant from Chicago and Detroit. Therefore, Mr. Dewey is not the brakeman's nearest neighbor, as he lives in Detroit. Therefore, Mr. Huey is the brakeman's nearest neighbor, living somewhere between Chicago and Detroit. Now we are tempted to assume that Mr. Louie lives in Chicago, but we don't have to. By statement number 6, we know that SOME passenger lives in Chicago, and we've placed two of the passengers, so therefore Mr. Louie must live in Chicago. By statement number 6 again, we know that Louie is the brakeman. Finally, we know that Huey is not the fireman, and he can't be the brakeman, since Louie is, so Huey must be engineer and Dewey the fireman. $\Rightarrow A$	30) $\frac{{}_S C_V}{{}_R C_U}\cdot T=\frac{{}_5 C_4}{{}_6 C_6}\cdot 8=40 \Rightarrow D$