

FAMAT State Convention 2001

For all questions, answer E. "NOTA" means none of the above answers is correct.

1. What is the slope of the line through the points (0,5) and (4,2)?

- A. -4/3 B. -3/4 C. 4/3 D. 3/4 E. NOTA

2. If the function f is everywhere continuous and has the values $f(2001) = 2001$ and $f(2002) = 2002$, then which answer below is possible as a necessary consequence of the Intermediate Value Theorem?

- A. $f(2002.5) = 2002.5$ B. $f(2001.5) = 2002.5$
 C. $f(2002.5) = 2001.5$ D. $f(2001.5) = 2001.5$
 E. NOTA

3. If $F = \int_0^{\pi} x \, dx$ and $G = \int_0^{\pi} \sin x \, dx$, then what is the value of FG ?

- A. π B. 2π C. 1 D. π^2 E. NOTA

4. A 45-caliber bullet fired straight up from the surface of the moon has the velocity $v(t) = 200 - 1.55t$ meters per second after t seconds. Approximately how many seconds (to the nearest second) would it take the bullet to get back down to the surface?

- A. 258 B. 129 C. 65 D. 516 E. NOTA

5. A bus will hold up to 60 people. The number x of people per trip who use the bus is related to the fare charged (p dollars) by the law $p = [3 - (x/40)]^2$. If the total revenue is given by the function $r(x)$, and the marginal revenue is given by dr/dx , then what fare corresponds to a marginal revenue of zero?

- A. \$0.00 B. \$3.00 C. \$9.26 D. \$40.00 E. NOTA

6. Which of the following expressions completes the function $f(x)$ below and makes it differentiable in the entire domain $x \in (-\infty, \infty)$?

$$f(x) = \begin{cases} 0 & x \leq 0 \\ \text{---} & 0 < x < \pi \\ 1 & x \geq \pi \end{cases}$$

- A. $(\sin x) + (x/\pi)$ B. $(1 - \cos x)/2$
 C. $-(x \cos x)/\pi$ D. $(\cos x)/2$ E. NOTA

7. Two curves are said to be orthogonal at a point of intersection if their tangents at that point cross at right angles. For the curves $2x^2 + 3y^2 = 5$ and $y^2 = x^3$, at which of the following points are the curves orthogonal?

- I. (1,1) II. (1,-1) III. (9,27) IV. (-9,27)
 A. I and II only B. III and IV only
 C. I, II, and III only D. I, II, III, and IV
 E. NOTA

8. Suppose the earth were a perfect sphere and that we determined its radius to be 6371.4 ± 0.16 km. What effect would the tolerance of ± 0.16 km have on the estimate of the earth's surface area? Choose the state below that most closely matches the possible change in the surface area for a change in radius of 0.16 km.

- A. Delaware (5060 km²) B. Maryland (25320 km²)
 C. Florida (139850 km²) D. Texas (678350 km²)

9. The four points $(-2, -0.5)$, $(0, 1)$, $(1, 2)$, and $(3, 3)$ are observed to lie more or less close to a line $y = mx + 1$. What value of m makes the sum $(y_1 - mx_1 - 1)^2 + (y_2 - mx_2 - 1)^2 + (y_3 - mx_3 - 1)^2 + (y_4 - mx_4 - 1)^2$ a minimum, where $(x_1, y_1), \dots, (x_4, y_4)$ are the coordinates of the given points?

- A. 5/7 B. 14/57 C. 9/4 D. 4/13 E. NOTA

10. An isosceles triangle is drawn with its vertex at the origin, its base parallel to and above the x -axis, and the vertices of its base on the curve $12y = 36 - x^2$. What is the area of the largest such triangle?

- A. $\sqrt{3}$ B. $\sqrt{3}/2$ C. $4\sqrt{3}$ D. $2\sqrt{3}$ E. NOTA

11. A circular sector has a perimeter of 100 meters. What is the maximum area that this sector can have (in square meters)?

- A. 100 B. 125 C. 468.75 D. 625 E. NOTA

12. For oscillations of small amplitude, the relation between the period T and the length L of a simple pendulum may be approximated by the equation $T = 2\pi\sqrt{L/g}$ where g is the acceleration due to gravity. When the temperature θ changes, the length either increases or decreases at a rate that is proportional to L (i.e., $dL/d\theta = kL$, where k is a proportionality constant). What is the rate of change of the period with respect to temperature?

- A. $\frac{\pi k}{2}\sqrt{L/g}$ B. $2\pi k\sqrt{L/g}$
 C. $kT/2$ D. kT E. NOTA

13. Graphing $f(x) = x^2 - 1 - 0.5\sin x$ suggests that the function has a root near $x = 1.25$. Use one application of Newton's method to improve this estimate. That is, start with $x_0 = 1.25$ and find x_1 (to the nearest 5 decimal places).

- A. 0.97422 B. 1.21243
 C. 1.28757 D. 1.52578 E. NOTA

14. Economists often use the expression "rate of growth" in relative rather than absolute terms. For example, in a given industry, let $u = f(t)$ be the number of people in the labor force at time t . (Treat this function as though it were differentiable even though it is an integer-valued step function.)

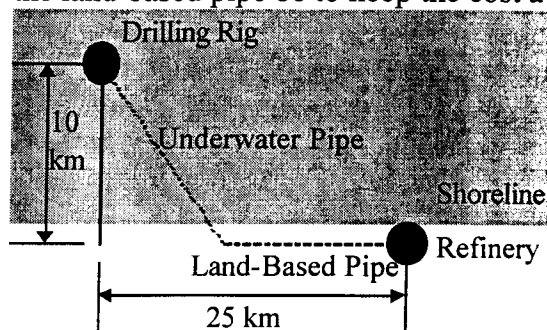
Let $v = g(t)$ be the average production per person in the labor force at time t . The total production is then $y = uv$. If the labor force is growing at the rate of 4% per year, and the production per worker is growing at the rate of 5% per year, then what is the rate of growth of the total production?

- A. 0.2% B. 1% C. 9% D. 20% E. NOTA

15. A body is moving with a velocity of 16 ms^{-1} when it is suddenly subjected to a deceleration. The deceleration is proportional to the square root of the velocity, and the body comes to rest in 4 seconds. How many meters does the body travel from when the deceleration starts to when the body comes to rest?

- A. 16 B. 8 C. $64/3$ D. $32/3$ E. NOTA

16. A drilling rig, 10 km offshore, is to be connected by pipe to a refinery on shore, 25 km down the coast from the rig, as shown in the figure below. If underwater pipe costs \$50,000/km and land-based pipe costs \$30,000/km, how many kilometers long should the land-based pipe be to keep the cost a minimum?



- A. 7.5 B. 10 C. 17.5 D. 15 E. NOTA

17. With approximately what speed (in ms^{-1}) does a diver enter the water after diving from a 10-meter platform? (Use $g = 9.8 \text{ ms}^{-2}$ for the acceleration of gravity and assume the diver has zero velocity when he starts the dive and that his motion is only vertical)

- A. 1.4 B. 2.5 C. 9.8 D. 14 E. NOTA

18. The velocity of a particle moving back and forth on a line is $v = ds/dt = 6\sin 3t \text{ m s}^{-1}$ for all t . If $s = 0$ when $t = 0$, then what is the total distance travelled by the particle from $t = 0$ to $t = \pi \text{ sec}$?

- A. 12 B. 8 C. 4 D. 6 E. NOTA

19. Suppose a wholesaler receives a shipment of 1200 cases of chocolate bars every 30 days. The chocolate is sold to retailers at a steady rate; and x days after the shipment arrives, the inventory of cases still on hand is $I(x) = 1200 - 40x$. What is the average daily holding cost for the chocolate if the cost of holding one case is 5 cents a day (Assume x and $I(x)$ are continuous)?

- A. \$15 B. \$30 C. \$45 D. \$60 E. NOTA

20. Using differentials beginning at $\sqrt{64}$, what is the approximate value of $\sqrt{65}$? Round answer to the nearest 4 decimal places.

- A. 8.0623 B. 8.0624 C. 8.0625 D. 8.0626
 E. NOTA

21. What is the approximate value of the integral $\int_0^4 \sqrt{y} dy$ when you use Simpson's Rule with $n = 6$?

Round your answer off to the nearest 2 decimal places.

- A. 5.29 B. 5.31 C. 5.32 D. 5.33 E. NOTA

22. A 60-gram tennis ball was served at 60 ms^{-1} (about 134 mph). How many Newton-meters of work were done on the ball to get it to this speed (Remember that $1 \text{ Newton} = 1 \text{ kgms}^{-2}$)?

- A. 216 B. 108 C. 54 D. 27 E. NOTA

23. Evaluate the following definite integral:

$$\int_1^{64} \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$$

- A. $-9 + 16 \ln 3$ B. $9 + 16 \ln(2/3)$
C. $-11 + 6 \ln 3$ D. $11 + 6 \ln(2/3)$ E. NOTA

24. What is the area of the infinite region in the first quadrant that lies between the curves $y = \sinh x$ and $y = \cosh x$?

- A. 2 B. 4 C. π D. Infinity E. NOTA

25. The vertex angle of a cone is the acute angle made between the lines containing the height of the cone and a slant height of the cone. For what vertex angle does a cone whose slant height is 3 m enclose the largest possible volume?

- A. $\cos^{-1}(\sqrt{3}/3)$ B. $\cos^{-1}(\sqrt{2}/2)$
C. $\tan^{-1}(\sqrt{6}/3)$ D. $\tan^{-1}(\sqrt{2})$ E. NOTA

26. The base of a solid is the circle $x^2 + y^2 = 9$. Each section of the solid cut by a plane perpendicular to the x -axis and parallel to the y - z plane is a square with one edge in the base of the solid. What is the volume of the solid?

- A. 36π B. 144π C. 36 D. 144 E. NOTA

27. The velocity of a particle moving along the x -axis is proportional to x . At time $t = 0$ the particle is located at $x = 2$ and a time $t = 10$ it is at $x = 4$. What is its position at $t = 5$?

- A. $\sqrt{2}/2$ B. $\sqrt{2}$ C. $2\sqrt{3}$ D. $\sqrt{3}$ E. NOT

28. A hard-boiled egg at 99°C is put in a sink of 19°C water to cool. After 5 minutes, the egg's temperature found to be 39°C . Assuming that the water has not warmed, how many more minutes will it take the egg reach room temperature (20°C)?

- A. $5 + 5 \log_4 5$ B. $5 \log_4 5$
C. $10 + \ln(1.25)$ D. $15 + \ln(1.25)$ E. NOTA

29. The volume of a spherical flask with an interior radius of 10 cm is to be increased by about 1 cm^3 by eating away the interior with hydrofluoric acid. Assuming the acid eats into the interior surface at a uniform 0.1 mm/hr and that the flask is not eaten entirely through, how many seconds should the flask be kept full of acid (to the nearest second)?

- A. 286 B. 314 C. 100 D. 464 E. NOTA

30. A triangle has sides of length x , $x + 1$, and $x + 2$. The area of a circle inscribed in this triangle is defined by the function $A(x)$. At what value(s) of x , if any, does the curve $A(x)$ have an inflection point?

- A. $2\sqrt{3}$ B. $-1 \pm 2\sqrt{3}$ C. $-1 + 2\sqrt{3}$
D. No inflection points E. NOTA