

Alpha Applications
2001 FAMAT State Convention

NOTA: None of these answers.

1. At what final velocity must a 100 kg football player run to dash 40 yards in 4.4 seconds (starting from rest and with constant acceleration)? Round to the nearest 10^{th} .
a) $54.6 \frac{ft}{s}$ b) $27.3 \frac{ft}{s}$ c) 620 slugs d) $6.20 \frac{ft}{s^2}$ e) NOTA
2. Calculate the smaller angle formed by the hands of a clock at 2:32.
a) 116° b) 124° c) 88° d) 12° e) NOTA
3. Many ancient cultures used a counting system based on the number 6 (in contrast to our base 10 system). How many days would one of the ancient astronomers say are in one “leap” year?
a) 1405 b) 365 c) 366 d) 1410 e) NOTA
4. Another astronomer (using the base 10 system) discovered the eccentricity of Pluto’s orbit to be roughly 0.81. Assuming a semi-minor axis of length 2.7 billion miles, what would be the area swept out by this planet in one orbit?
a) 4.2×10^{12} square miles b) 8.9×10^6 square miles c) 3.9×10^{19} square miles
d) 2.3×10^{88} square miles e) NOTA
5. A cylindrical tree with a diameter of two meters with its bark removed is cut down to form cubes. What is the length of an edge of the largest possible cube that can be made?
a) $2\sqrt{2}$ meters b) 2 meters c) $\sqrt{2}$ meters d) 1 meter e) NOTA
6. How much Energy is theoretically contained within an object with Mass of 1 kg?
(hint: speed of light (C) = $2.99 \times 10^8 \frac{m}{s}$)
a) 2.99×10^8 joules b) 1 joule c) 1.12×10^{-17} joules d) 3.34×10^{-9} joules e) NOTA
7. If a “word” consists of letters grouped together, how many distinguishable four-letter “words” could a politician make from the characters in ‘Florida’?
a) 35 b) 24 c) 34^{24} d) 840 e) NOTA
8. Snell’s Law for refracting light from air is follows $\frac{\sin(\Theta_i)}{\sin(\Theta_r)} = k$. At what index of refraction (k) is glass if the angle of incidence is 52° and the angle of refraction is 31.3° ?
a) 1.52 b) 0.658 c) 1.00 d) 1.33 e) NOTA

9. The augmented matrix $\left\langle \begin{array}{ccc|c} 1 & -3 & 3 & 1 \\ 2 & 0 & -3 & -1 \end{array} \right\rangle$ yields...

- a) no solutions b) only 1 solution c) infinite solutions d) can't be determined e) NOTA

10. Tesla, the evil genius, finds that the current in an AC circuit follows the function of time:

$$f(t) = \frac{\csc(t) \cdot \cos(2t)}{(\sin^2(t) + \sin(t) \cdot \cos(t)) \cdot (1 + \cot^2(t))}.$$

Please simplify $f(t)$ thereby qualifying yourself for the Nobel Prize in trig substitution. Assume $f(t)$ to be continuous at all points.

- a) $\sin(t)$ b) $\sin(t) - \cos(t)$ c) $\cos(t) - \sin(t)$ d) $\cos(t)$ e) NOTA

11. 2001 has 3 prime factors. How many factors are there of 2001^2 ?

- a) 6 b) 9 c) 3 d) 36 e) NOTA

12. Which of the following is a vector quantity?

- I. Force II. Mass III. Speed IV. Acceleration V. Energy VI. Pressure

- a) I, III, IV, VI b) I, II, III, V c) I, V, IV d) I, IV, VI e) NOTA

13. A wind blowing directly east at 10 m.p.h. must be taken into account by a helicopter intending to fly directly north. If the chopper's max speed in still air is 90 m.p.h., at what heading must the pilot take to fly directly north? Round to the nearest tenth.

- a) 6.4° west of north b) 6.3° east of north c) due north
d) 13.7° west of south e) NOTA

14. Two waves are emitted from a double slit and experience diffraction. They are said to be in phase when $d \cdot \sin(\Theta) = I \cdot m$ (for $m = 0, 1, 2, \dots$). For how many ordered pairs (m, Θ) does this equation hold if $d = 2\text{m}$ and $I = 0.5\text{m}$? ($0^\circ \leq \Theta \leq 90^\circ$)

- a) 0 b) 1 c) 4 d) 5 e) NOTA

15. $y_1 = 10 \cdot \sin(\mathbf{w} \cdot t)$, $y_2 = 8 \cdot \sin(\mathbf{w} \cdot t + 30^\circ)$, $y_3 = y_1 + y_2$. Calculate the amplitude of y_3 .

- a) 18 b) 17.4 c) 19.1 d) 16.9 e) NOTA

16. A 10 question test is scored in the following manner: 1 point for every correct answer, 0 points for every omission and -0.25 points for every incorrect answer beyond the second mistake. What is the lowest unachievable score (S) greater than 0? (n.b.: $4S=I$, where I is an integer.)

- a) 4.25 b) 5.5 c) 8.25 d) 7.25 e) NOTA

17. A basketball and a baseball are 12 and 4 inches in diameter, respectively. What is the maximum number of baseballs that can be arranged around the basketball such that each baseball is tangent to the basketball, at least one other baseball, and the ground?

- a) 6 b) 10 c) 8 d) 12 e) NOTA

18. Newton stated the universal law of gravitational force to be $F = G \cdot \frac{m_1 \cdot m_2}{r^2}$ where $G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$.

Allen and Mary's masses are 70.3 kg and 54.4 kg, respectively. Calculate the force of attraction betwixt those crazy kids when they stand 0.01 m apart.

- a) 0.0697 N b) 0.00255 N c) 0.324 N d) ?2.71 N e) NOTA

19. It takes 2 hours to drive from Gainesville to Tallahassee traveling at 75 m.p.h. Calculate the angular velocity of the wheels of a truck in radians per second if the wheel radius is 15 inches.

- a) $67 \frac{rad}{aec}$ b) $23 \frac{rad}{sec}$ c) $52 \frac{rad}{sec}$ d) $88 \frac{rad}{sec}$ e) NOTA

20. The Fibonacci sequence is defined as follows: let $t_1 = t_2 = 1$, $t_n = t_{n-1} + t_{n-2}$ for $n \geq 3$. The sum of the first n terms of this sequence is given by...

- a) t_{n+1} b) $t_{n+1} + 1$ c) $t_{n+2} - 1$ d) $t_{n+2} + 1$ e) NOTA

21. A spherical balloon's radius shrinks from 12 cm to 10 cm. What percent of the balloon's original volume was lost in the process?

- a) 57.9% b) 42.1 % c) 69.4 % d) 30.6 % e) NOTA

22. One of the corner sections of a football stadium has one seat in the first row, three seats in the second row, five seats in the third row, and so on. If this section of the stadium will hold 1600 people when full with one person per seat, how many people can sit in the last row, one person per seat?

- a) 79 b) 101 c) 131 d) 159 e) NOTA

23. Determine the Cartesian coordinates of the foci of the conic section determined by the polar equation:

$$r = \pm \sqrt{\sec(2 \cdot \Theta)}$$

- a) $(0, \pm \sqrt{2})$ b) $(\pm \sqrt{2}, 0)$ c) $(0, \mp \sqrt{3})$ d) $(\mp \sqrt{3}, 0)$ e) NOTA

24. A regular octagon has sides of length s. Calculate the area of the octagon in terms of s.

- a) $s^2(1 + \sqrt{2})$ b) $4s^2(2 + \sqrt{2})$ c) $2s^2(1 + \sqrt{2})$ d) $s^2(2 + \sqrt{2})$ e) NOTA

25. At what angle should a projectile be fired if it is to travel a maximum horizontal distance in still air?

- a) 22.5° b) 30° c) 60° d) 45° e) NOTA

26. A bug starts at the origin and moves 1 space to the right. It then turns 90° counterclockwise and proceeds to move $\frac{1}{2}$ the distance it previously moved and turns the same way again. How far from the origin will the bug be after it has continued through an infinite sequence of moves and turns in this pattern?

- a) $2\sqrt{5}$ b) $\frac{\sqrt{5}}{5}$ c) $2\frac{\sqrt{5}}{5}$ d) $\frac{\sqrt{10}}{5}$ e) NOTA

27. Evaluate $\lim_{h \rightarrow 0} \frac{e^{(x+h)} - e^x}{h}$

- a) e^x b) $2e^x$ c) 0 d) 1 e) NOTA

28. For a genetics experiment, a biology class has a sample of 100 flies. With a constant supply of food and space, the fly population is modeled by $p(t) = 100e^{1.63 \sin(0.139 \cdot t)}$. What is the maximum population, and when is it first achieved?

- a) 510 flies, 11.3 days b) 255 flies, 22.6 days c) 510 flies, 22.6 days
d) 255 flies, 11.3 days e) NOTA

29. A radioactive isotope decays according to the equation $\frac{1}{[A_t]} = k \cdot t + \frac{1}{[A_0]}$. Calculate the half life in terms of k and $[A_0]$.

- a) $\frac{k}{[A_0]}$ b) $\frac{[A_0]}{2k}$ c) $k[A_0]$ d) $\frac{1}{k[A_0]}$ e) NOTA

30. A carpenter is building a walkway over a garden pool that is shaped like a parallelogram. If the walkway is to span the pool above the shortest diagonal, find the length of that diagonal, when the sides of the pool are 12 meters and 16 meters with one angle of the parallelogram with a measure of 120 degrees.

- a) $4\sqrt{13}$ meters b) $11\sqrt{2}$ meters c) $6\sqrt{7}$ meters d) $5\sqrt{11}$ meters e) NOTA