

### Advanced Calculus Topic Test

FAMAT State Convention 2001

NOTA stands for None of the Above.  
Select NOTA if choices A through D are incorrect.

1. Find the arc length of

$$y = \frac{x^3}{12} + \frac{1}{x} \text{ from } x = 1 \text{ to } x = 2.$$

A.  $\frac{1}{12}$

B. 2

C.  $\frac{13}{12}$

D.  $12^{\frac{3}{2}} - 8^{\frac{3}{2}}$

E. NOTA

2. Evaluate  $\int_0^{\frac{p}{2}} (e^x \cdot \cos(x)) dx$

A.  $\frac{e^{\frac{p}{2}} - 1}{2}$

B.  $e^{\frac{p}{2}} - 1$

C.  $\frac{e^{\frac{p}{2}} + 1}{2}$

D.  $\frac{e^{\frac{p}{2}}}{2}$

E. NOTA

3. Evaluate  $\int_0^{\frac{p}{4}} (\tan^2(x) \cdot \sec^4(x)) dx$

A. 2

B.  $\frac{p^4}{128} + \frac{p^2}{16}$

C.  $\frac{6}{5}$

D.  $\frac{8}{15}$

E. NOTA

4. Evaluate  $\int_2^3 \frac{2}{x^2 - 1} dx$ .

A.  $\ln \frac{8}{3}$

B.  $\ln \frac{5}{3}$

C.  $\ln \frac{2}{3}$

D.  $\ln \frac{1}{3}$

E. NOTA

5. Evaluate  $\lim_{x \rightarrow 0} (1 + 3x)^{\frac{1}{2x}}$

A. 4.5

B.  $e^{\frac{3}{2}}$

C.  $e^{\frac{4}{3}}$

D.  $\frac{3}{2}$

E. NOTA

6. Evaluate  $\int_1^{\infty} \frac{x}{(1+x^2)^2} dx$

A. 0.5

B. 0.25

C. 0.2

D.  $\infty$

E. NOTA

7. The Maclaurin series for  $f(x)$  is given

by  $1 + \frac{x}{2!} + \frac{x^2}{3!} + \frac{x^3}{4!} + \dots + \frac{x^n}{(n+1)!} + \dots$ . Which

of the following is  $f(x)$ ?

A.  $\frac{e^x}{x}$

B.  $e^x$

C.  $e^x - 1$

D.  $\frac{e^x - 1}{x}$

E. NOTA

8. Which of the given series converge?

I.  $\sum_{n=1}^{\infty} \frac{\cos(np)}{\sqrt{n}}$

II.  $\sum_{n=1}^{\infty} \frac{\sin(\sqrt{n})}{\sqrt{n^3+1}}$

III.  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

- A. I and II only  
 B. II and III only  
 C. I and III only  
 D. I, II, and III  
 E. NOTA

9. Find the interval of convergence of  $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$ .

- A.  $(-1,1)$   
 B.  $[-1,1)$   
 C.  $(-1,1]$   
 D.  $\infty$   
 E. NOTA

10. Which of the following is a power series representation for  $\frac{1}{(1+x)^2}$  for  $|x| < 1$ ?

- A.  $1 - 2x + 3x^2 - \dots + (-1)^{n+1} n \cdot x^{n-1} + \dots$   
 B.  $-1 + 2x - 3x^2 + \dots + (-1)^n n \cdot x^{n-1} + \dots$   
 C.  $x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^n \frac{x^{n+1}}{n+1} + \dots$   
 D.  $1 - x + x^2 - x^3 + \dots + (-1)^n x^n + \dots$   
 E. NOTA

11.  $\sum_{n=1}^{\infty} (2^{-n-1} - 3^{-n-1}) =$

- A.  $1/2$   
 B.  $1/3$   
 C.  $1/5$   
 D.  $1/6$   
 E. NOTA

12. Write the first three terms of the MacLaurin Series for  $\sin(x^2)$  and use them to approximate the value of  $\int_0^1 \sin(x^2) dx$  to the nearest hundred-thousandth.

- A. 0.31026  
 B. 0.31027  
 C. 0.31028  
 D. 0.31029  
 E. NOTA

13. A curve has parametric equations  $x = t^3 - 3t, y = t^2 - 5t - 1$  for  $t > 0$ . Find the slope of the tangent line to the curve at  $(-2, -5)$ .

- A.  $-1$   
 B.  $-\frac{1}{2}$   
 C.  $1$   
 D. undefined  
 E. NOTA

14. Find the area of the region bounded by one loop of the graph of  $r^2 = 4\cos(2\theta)$ .

- A. 1  
 B. 2  
 C. 4  
 D.  $2p$   
 E. NOTA

15. Find the length of one arch of the cycloid which has parametric equations  $x = t - \sin t$  and  $y = 1 - \cos t, 0 \leq t \leq 2p$ .

- A.  $2p$   
 B. 4  
 C. 6  
 D. 8  
 E. NOTA

16. Find the slope of the line tangent to

$$r = 4 \sin(3q) \text{ at } q = \frac{P}{4}.$$

- A.  $-6\sqrt{2}$   
 B.  $\frac{\sqrt{2}}{2}$   
 C.  $\frac{1}{2}$   
 D.  $\frac{1}{4}$   
 E. NOTA

17. Find the first 3 terms of the Taylor Series

for  $\tan(x)$  centered at  $\frac{P}{4}$ .

- A.  $1 + 2\left(x - \frac{P}{4}\right) + 2\left(x - \frac{P}{4}\right)^2$   
 B.  $1 + 2\left(x - \frac{P}{4}\right) + 4\left(x - \frac{P}{4}\right)^2$   
 C.  $1 + 2x + 4x^2$   
 D.  $1 + \sqrt{2}\left(x - \frac{P}{4}\right) + 2\left(x - \frac{P}{4}\right)^2$   
 E. NOTA

18. Which vector will be tangent to the given vector-valued function if drawn from the point

$(1,1)$  on the curve?  $\vec{r}(t) = \langle e^{2t}, e^{-t} \rangle$

- A.  $\langle 2, -1 \rangle$   
 B.  $\langle -2, 1 \rangle$   
 C.  $\langle 1, -2 \rangle$   
 D.  $\langle -1, 2 \rangle$   
 E. NOTA

19.  $\int_0^1 \left( 1 + 2x + 2x^2 + \frac{8x^3}{3!} + \frac{4x^4}{3!} + \frac{32x^5}{5!} + \dots \right) dx =$

- A.  $e^2 - 1$   
 B.  $e^2$   
 C.  $e$   
 D.  $\frac{e^2 - 1}{2}$   
 E. NOTA

20. The position of a particle moving in a plane is given by the vector-valued function

$$\vec{r}(t) = \langle 2t, 4t^2 + 1 \rangle. \text{ Find its speed at } t = 1.$$

- A.  $\langle 2, 8 \rangle$   
 B. 68  
 C.  $\langle 2, 5 \rangle$   
 D.  $\sqrt{29}$   
 E. NOTA

21. Which of the following is a solution to the differential equation

$$2xy + 6x + (x^2 - 4) \frac{dy}{dx} = 0?$$

- A.  $y = \frac{C}{x^2 - 4} - 3$   
 B.  $y = \frac{1}{x^2 - 4} + C$   
 C.  $y = -\ln|x^2 - 4| + C$   
 D.  $y = C \cdot \ln|x^2 - 4|$   
 E. NOTA

22. Solve the differential equation

$$x^2 \frac{dy}{dx} + 5xy + 3x^5 = 0, x \neq 0.$$

- A.  $y = -\frac{x^4}{3} + Cx^5$   
 B.  $y = -3x^3 + C$   
 C.  $y = -\frac{x^4}{3} + C$   
 D.  $y = -\frac{x^4}{3} + \frac{C}{x^5}$   
 E. NOTA

23. If  $g(x) = \int_1^x \frac{1}{t} dt$  for  $x > 0$ , then  $g(x^3) =$

- A.  $(g(x))^3$   
 B.  $3x^2 \cdot g(x)$   
 C.  $3g(x)$   
 D.  $g\left(\frac{1}{x^3}\right)$   
 E. NOTA

24. If  $\int_0^x \frac{1 - \cos t}{t^2} dt = \sum_{k=1}^{\infty} c_k x^k$ , then  $c_9 =$

(Note: This is an improper integral that does not converge to a power series.)

- A.  $-\frac{1}{9!}$   
 B.  $\frac{1}{9 \cdot 10!}$   
 C.  $-\frac{1}{9 \cdot 10!}$   
 D.  $\frac{1}{9!}$   
 E. NOTA

25. Evaluate  $\int_0^4 \frac{1}{(x-3)^2} dx$ .

- A.  $-\frac{4}{3}$   
 B.  $\frac{4}{3}$   
 C.  $\frac{2}{3}$   
 D.  $\frac{3}{2}$   
 E. NOTA

26. Evaluate  $\int_3^{\infty} \frac{1}{x^2 - 1} dx$

- A.  $\infty$   
 B.  $\ln \frac{3}{2}$   
 C.  $\ln 2$   
 D.  $\ln \sqrt{2}$   
 E. NOTA

27. The third degree Taylor polynomial for  $\ln x$  centered at  $a = 1$  is used to evaluate  $\ln(1.3)$ . What is the maximum error obtained using the Lagrange form of the remainder?

- A. 0.002025  
 B. 0.002005  
 C. 0.001843  
 D. 0.001636  
 E. NOTA

28. Given  $x(t) = t^3 - 2$  and  $y(t) = 3t^2$ .

Find  $\frac{dy}{dx}$  as a function of  $x$ .

- A.  $\frac{2}{x^3 - 2}$   
 B.  $\frac{\sqrt[3]{x+2}}{2}$   
 C.  $\frac{2}{\sqrt[3]{x+2}}$   
 D.  $\frac{2}{x}$   
 E. NOTA

29. Evaluate  $\int_0^1 \frac{x^2}{x^6 + 4} dx$

- A.  $\ln 5$   
 B.  $\frac{\arctan\left(\frac{1}{2}\right)}{6}$   
 C.  $\frac{\ln 5}{2}$   
 D.  $\frac{\arctan\left(\frac{1}{2}\right)}{3}$   
 E. NOTA

30. The region bounded by the graphs of

$$y = \frac{1}{(x-1)(4-x)}, y = 0, x = 2, \text{ and } x = 3$$

is revolved about the  $y$ -axis. Find the volume of the resulting solid.

- A.  $\frac{10p \ln 2}{3}$   
 B.  $\frac{5 \ln 2}{3}$   
 C.  $2p \ln 2$   
 D.  $\ln 2$   
 E. NOTA