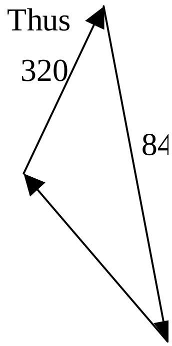
2001 STATE CONVENTION – TRIGONOMETRY TOPIC TEST SOLUTIONS

1. Use the definition of reference angle. (The acute angle between the terminal side of the angle in question and the x-axis). In this case, 22^o has a R.A. of 22^o, the only choice that does is 202^o (180 + 22). [C].

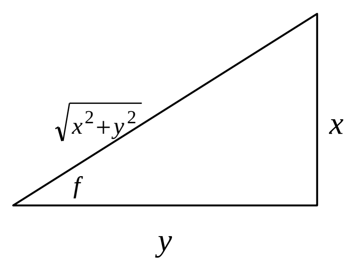
2. Apply $1-\sin^2 x=\cos^2 x$. The "*sec(x)*" terms cancel, as will one *cos(x)* (numerator) and the denominator *cos(x).* Complete with identities to find *csc(x),* or [B].

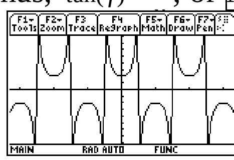
3. Draw a diagram as shown below. Convert the given speeds/times into distances using $d = rt$. The 841 side is 15^o E of S and using angle theorems, the 320 side is 29^o W of S. Combine these to find the enclosed angle to be 44^o. Now apply law of cosines. $c^2=a^2+b^2-2ab\cos q=320^2+841^2-2(320)(841)\cos(44^o)$; c = 650. Thus the total distance is 650+320+841 = 1811, [C].



4. Phase shift is $-\frac{-p}{5}$ or $\frac{p}{5}$ and the period is $\frac{2p}{5}$. Their sum is $\frac{3p}{5}$, [A].

5. Draw a representative triangle as shown. Thus, $\tan(f)=\frac{x}{y}$, or [A].



6. Shown is the graph of the secant function. 

   The range of the function is all reals greater than or equal to 1 or less than or equal to negative one. Also expressed as $(-\infty,-1]\ U\ [-1,\infty)$, [A].

7. Setup cross-product matrix tool. $\begin{vmatrix} i & j & k \\ -3 & -5 & 6 \\ 3 & 2 & 5 \end{vmatrix}$. q x p = (-25-12)*i* + -(-15-18)*j* + (-6+15)*k* = –37*i* + 33*j* + 9*k*, [C].

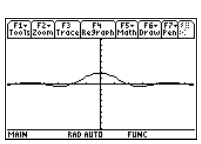
8. Use $s=rq$, $\frac{5p}{3}=r(\frac{p}{3})$, Thus r = 5, [B].

9. Multiply by the conjugate, (2-2*i*) to get $\frac{6-102i}{8}=\frac{3}{4}+\frac{-51}{4}i$. $\frac{3}{4}+\frac{-51}{4}=-12$, [A].

10. As *x* increases w/o bound, the "-6x", "cos(x)", "sin(x)", and "5" terms become negligible. The resulting limit is the quotient of coefficients, or $\frac{5}{-1}$ or –5, [A].

11. Factor the equation into $(\cos(b)+\sin(b))(\cos(b)+\sin(b))=0$, Thus $\cos(b)+\sin(b)=0$; $\cos(b)=-\sin(b)$. This holds true at $\frac{3p}{4}$ *and* $\frac{7p}{4}$. Their sum is $\frac{5p}{2}$, or [D].

12. Use the right hand rule to rotate ***p*** onto ***q*** and the resulting direction is +k, or [A].

13. Graph is shown. 

The function oscillates away from 0, but approaches 1 at *x* = 0, so [B].

14. Apply the identity $\cos(f-g)=\cos(f)\cos(g)+\sin(f)\sin(g)$. $Sin(f)$ and $\cos(g)$ are given. Their opposites ($\cos(f)$ and $\sin(g)$) can be found using 1-$\sin^2$(x) = $\cos^2$(x). Expressed in A and B form, the result is $A\sqrt{1-B^2}+B\sqrt{1-A^2}$, or $\boxed{D}$.

15. First, find the acute angle formed with the *x*-axis, using $\tan(q)=m$; $\tan(q)=\frac{3}{5}$, $q\approx 30.963°$. Thus the angle formed with the *y*-axis is 90-30.963 or 59°, or $\boxed{D}$.

16. $Frequency=\frac{1}{Period}$; $F=\frac{1}{2}$, or $\boxed{B}$.

17. I. Use sum/Diff. Identities to reduce *sin*(ρ+δ) + *sin*(ρ - δ) to 2*sin*(ρ)*cos*(δ), TRUE. II. Numerator should be $\tan(r)-\tan(d)$, FALSE. III. False. Therefore only I is true, choice $\boxed{A}$.

18. Set the *x*- and *y*- equations equal to zero separately. $\begin{matrix}0=3+3\cos(t)\\0=2+2\cos(t)\end{matrix}$ and solve for "t". $p+\frac{3p}{2}=\frac{5p}{2}$, or $\boxed{C}$.

19. Apply the formula $A=\frac{1}{2}ab\sin(q)$; $21=\frac{1}{2}(7)(12)\sin(q)$; $\sin(q)=\frac{1}{2}$; $q=30°$, or $\boxed{C}$.

20. Convert the trip distance to inches, 5448960 inches. The tire turns 48 inches per revolution, so the tire will rotate a total of 113520 times. Each revolution is 2π radians, so the total number of radians is (2π)(113520) or 227040π, or $\boxed{C}$.

21. $r=\sqrt{x^2+y^2}$; $r=\sqrt{20a^2}$, $r=2\sqrt{5}a$. $\tan(q)=\frac{-4a}{2a}$, $q\approx 297°$. Thus $(2\sqrt{5}a, 297°)$ is correct, however due to the symmetric properties of polar coordinates, by negating the "*r*" term, the angle can be reduced and the answer is $\boxed{A}$.

22. I. Orthogonal (perpendicular) vectors have a dot product of zero, true. II. The dot product is commutative, true. III. Cross product is *not* commutative, false. Therefore I & II, or $\boxed{B}$.

23. As *x* increases w/o bound, arcsec(x) decreases approaching a horizontal asymptote of $\frac{p}{2}$. $\boxed{B}$.

24. Use the formula to determine q, $\cot(2q)=\frac{A-C}{B}$ for a conic of the form Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0. Therefore, $\cot(2q)=\frac{4-5}{-\sqrt{3}}=\frac{1}{\sqrt{3}}$. Thus $q=30°$, or $\boxed{A}$.

25. The vertical shift required is p. For the horizontal shift, only $\frac{p}{2}$ is required. Thus $\frac{3p}{2}$, or $\boxed{C}$.

26. This is the beginning of the "long form" of the derivative for x^3, which can quickly be reduced to $3x^2$. Otherwise, expansion results in $\frac{3hx^2+3h^2x+h^3}{h}$; $3x^2+3h\ x+h^2$. Applying the limit yields $3x^2$ or $\boxed{D}$.

27. $\sec(p/4)=\sqrt{2}$, Thus becomes $\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\sqrt{2}}}}=x$; $\sqrt{2}^{x}=x$; $\ln(\sqrt{2}^{x})=\ln(x)$; $x\ln(\sqrt{2})=\ln(x)$; $x\ln(2)^{1/2}=\ln(x)$; $\frac{1}{2}\ln(2)=\frac{1}{x}\ln(x)$, thus x = 2, or $\boxed{C}$.

28. A one-to-one function must pass a <u>horizontal</u> line test. Only D passes, so $\boxed{D}$.

29. The summation becomes cos(π) + cos(2π) + cos(3π) + cos(4π) … which reduces even further to –1 + 1 + -1 … The first 100 terms all cancel out, reducing to -1, or $\boxed{A}$.

30. The fifth roots are given by $\sqrt[5]{2}\text{cis}\frac{p(4n+1)}{10}, i=0,1,2,3,4$ Note: $\text{cis}q=\cos q+i\sin q$) Hence the roots are $\sqrt[5]{2}\text{cis}\frac{p}{10}, \sqrt[5]{2}\text{cis}\frac{p}{2}, \sqrt[5]{2}\text{cis}\frac{9p}{10}, \sqrt[5]{2}\text{cis}\frac{13p}{10}, \sqrt[5]{2}\text{cis}\frac{17p}{10}$., A.

2001 STATE CONVENTION – TRIGONOMETRY TOPIC TEST ANSWERS

1C
2B
3C
4A
5A
6A
7C
8B
9A
10A
11D
12A
13B
14D
15D
16B
17A
18C
19C
20C
21A
22B
23B
24A
25C
26D
27C
28D
29A
30A