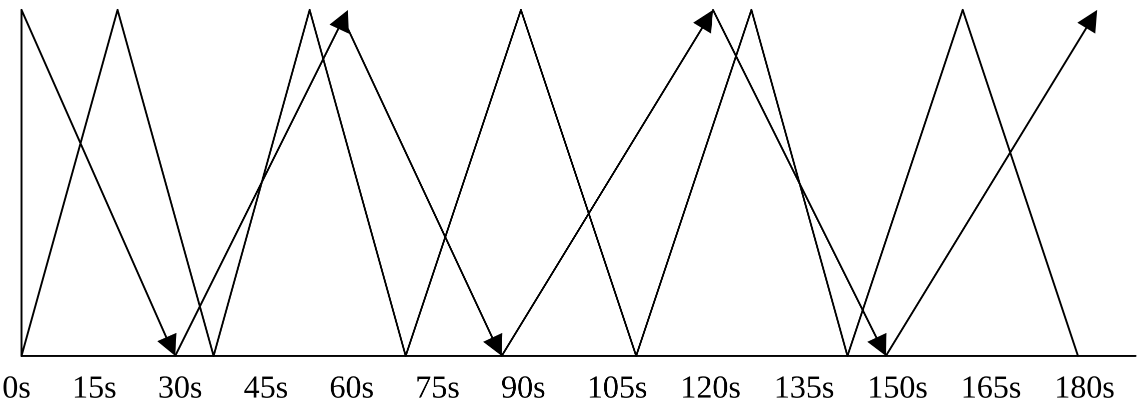
Theta Applications Solutions

1. Based on the definition of percentages and the fact that we are finding "the percent that 100 is greater than 64, 64 is in the denominator and 100(36) is in the numerator. This gives us a percentage of 56.3 when rounding to the nearest tenth. ***C***

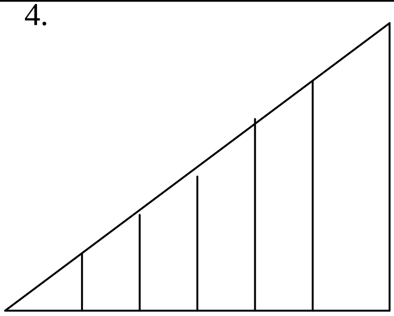
2. The first statement tells us that G = x + (x + 2) + (x + 4) + (x + 6), where x is an odd whole number. The second statement tells us that G = y + (y+2), where y is an odd whole number. The third statement tells us that G = z + (z + 1) + (z + 2) + (z + 3) + (z + 4), where z is a whole number. We now know that G = 4x + 12 = 2y + 2 = 5z + 10. Looking at this, we can tell that G must be divisible by 4 and 5. The first positive perfect square that achieves this is 100. ***E***

3. Let us graph the first three minutes of the swim for each swimmer



In the first 3 minutes, they pass each other 10 times. After these three minutes, they are back in their initial positions. Therefore, after 12 minutes, they pass each other 40 times. ***C***

4.



This is a potential outside drawing of what the fences would look like with 15 equal parts. We have on our hands in a series. Let L equal the length of AC. The largest parallel fence, for our purposes G(1), is solved by the following: $\frac{10}{L} = \frac{G(1)}{\frac{14}{15}L}$. Thus G(1)= $\frac{10(14)}{15}$ By similar logic, G(2)= $\frac{10(13)}{15}$. This process continues until the last term of the series, which is G(14)= $\frac{10(15-(15-1))}{15} = \frac{10}{15}$. Thus, we now have an arithmetic series whose first term is G1, whose last term is G(14), with 14 terms. Thus, the sum of the lengths of any number of parallel fences is 5(14)=70 ***A***

5. Let the first number be 10t + u, where t is the tens digit and u is the units digit, Therefore, the second number is 10u + t as the digits are reversed. $(10t + u)^2 - (10u + t)^2 = 99t^2 - 99u^2$. This is equal to 99(t-u)(t+u), which satisfies all conditions but ***B***

6. The first trip satisfies r t_1=50, where r is the rate of travel and t_1 is the time involved. The second trip satisfies 3r t_2 =300, where t_2 is the time for the second trip. Dividing the second equation by the first, we get that $\frac{t_2}{t_1} = 2$. ***B***

7. We must establish what we know can/cannot occur. The bag with 34 marbles cannot be the separate one, because at most Mary Jane would have 69 marbles and Kelly would have 37. BY similar logic, Mary Jane must have taken the 34 marble bag. Additionally, we know that since Mary Jane's total is twice Kelly's , her total must be even. Therefore, since there is only one other even bag, Mary Jane must have two of the four odd bags. By narrowing down the possibilities, an easy guess and check shows us that 23 is the only bag remaining, with Mary Jane taking the 34, 19 ,and 25 marble bags, and Kelly take the 18 and 21. **D**

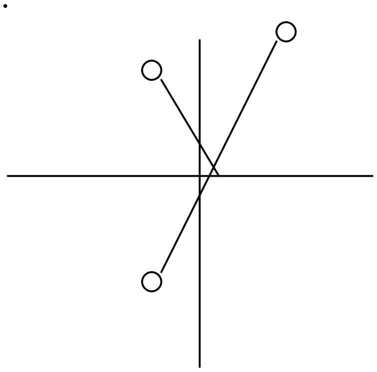
8. Let x equal the thickness of an algebra book. Let z equal the thickness of a geometry book. Let L equal the length of the shelf. Thus we have three conditions. Dx + Uz =L ; Bx + Yz = L ; Ax=L. We must solve the first two conditions for x and z as a system. Solving for x, we find that X= $\frac{L(Y-U)}{DY-BU}$. Subbing this value of x into our third condition we get that A= $\frac{DY-BU}{Y-U}$. ***C***

9. Let A= Michael's rate. Let B = Marion's rate. Let C= C.J.'s rate. Because rate times time equals distance, we can establish three equalities. $\frac{g}{A} = \frac{g-20}{B}, \frac{g}{B} = \frac{g-10}{C}, \frac{g}{A} = \frac{g-28}{C}$. Seeing a similarity between equality 1 and 3, we also know that $\frac{g-28}{C} = \frac{g-20}{B}$. Subtracting the second equality from this new equation, we find that 9B=10C. After substituting for B that B=$\frac{10}{9}C$ into the second equality, we solve for g, which equals 100. The tenths digit of ln 100 is 6. ***C***

10. We want the minimum value, and thus we want the highest sum of digits and the lowest number itself. This occurs at 189, where $\frac{189}{18} = 10.5$. The digits must be different. ***D***

11. There is a 50 percent chance that it will rain, regardless of whether Kelly washes his car. ***E***

12.



We want to find the shortest combined distances from (8,15) and (-2,9) to the x axis. However, it is shown in the diagram that a line reflecting off the x-axis and up to (-2,9) from (8,15) is just line a line from (8,15) to (-2,-9). Thus, we simply need to find the intersection of the line containing (8,15) and (-2,-9) with the x-axis. Knowing we have a slope of 2.4, the line has equation y+9=2.4(x+2). Since the x-axis has y equaling zero, we know that the line crosses the x-axis at 1.75. Thus, the fraction is $\frac{7}{4}$, so 7 + 4 = 11. ***D***

13. Duke has already given up 48 + 60 + 61 + 68 + 77=314 points. In order for Duke to average a minimum of 60 points, it simply must have an average of 59.5 (remember, we can round) over its 6 games. 59.5(6)-314=43. On the high end, Duke can also average $65.\bar{3}$ over its 6 games. $65.\bar{3}$(6)-314=78. 78-43 = 35. ***C***

14. This is a system. Let x equal the number of two point problems. Let y equal the number of three point problems. x + y =42. 2x + 3y=100. y=16, x =26 ***A***

15. Let A = the number of people for the Bill originally. Let B= the number of people for the Bill the second time. Let M=the positive margin of defeat from the first vote. We have the following equations: (150-A)-A=M B-(150-B)=2M, B=$\frac{7}{6}$(150-A). Multiplying the first equation by two and subtracting from the second equation and simplifying, we know B + 2A=225. Using the third equation with our new equation, we find that A=60 and B=105. Therefore, 60 voted for it the first time and 45 voted for it the second. 60-45 = 15 ***A***

16. This can be set up at a right triangle. He goes 1.5 rotations around a circle with radius $\frac{3}{p}$, so he has traveled $2p\frac{3}{p}\frac{3}{2}$=9. The ant has traveled halfway down a cylinder with height 24, so he has gone 12. We have a right triangle with legs 9 and 12. $\sqrt{9^2+12^2}$ =15. ***B***

17. We know all four cannot be false, because that makes the fourth statement true. Therefore, three statements must be false, with the third statement being true. ***D***

18. If 27 men built 9 houses, then each men builds $\frac{1}{3}$ of a house. If it takes 45 days, then each man contributes $\frac{1}{135}$ house per day. If 12 more houses need to be built in only 30 days, we must solve the equation $\frac{30x}{135}=12$, where x is the number of men needed. X = 58, so 58-27=31 ***A***

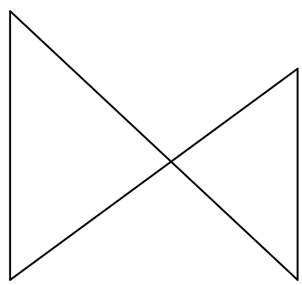
19. The price of 1 is x, the price of 2 is x + 3, 3 is x + 6, and so on. Therefore, the price of the 6 is x + 15 and 11 is x +30. We know that x + (x + 15) > x + 30, so x>15. The price of 8 is x + 21, so 8 is greater than 36 dollars. ***D***

20. By definition of average, they are the same. ***A***

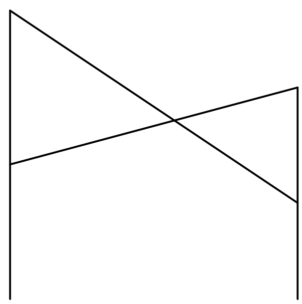
21. This is a set of two systems. Let D=Duke's score and N=North Carolina's Score. We know D= 4N and D= 3(N+3). Therefore N equals 9, and D equals 36. After North Carolina scores 3 more, Duke is winning by 24. ***B***

22. Let x equal the amount that Gregory will receive. Using rations we know that Orrin must receive $\frac{12x}{17}$, Rodney must receive $\frac{16x}{17}$, and Erin must receive $\frac{15x}{34}$. We also know that the total amount of money was \$1680, so x + $\frac{12x}{17}$ + $\frac{16x}{17}$ + $\frac{15x}{34}$ = \$1680. x = \$544. Therefore, Rodney receives \$512. The hundredths digit of ln 512 is 0. ***E***

23. This can be solved by simply finding the ratio of their heights. Allen is 5(12) + 10=70 inches. Shaq is 7(12)=84 inches. $\frac{70}{84} = \frac{5}{6}$. ***C***

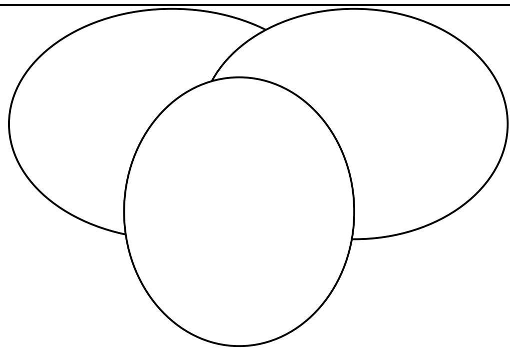


24. We have two intersecting lines. Let the bottom of the 120 foot tower be the coordinate (0,0). Therefore, the top of the 120 foot tower is the point (0,120). The bottom of the 80 foot tower is (100,0). The top of the 80 foot tower is (100,80). The equation of the first wire, which is from (0,0) to (100,80) is y=.8x. The second wire is from (100,0) to (0,120). The equation of the second wire is y= -1.2x+120. These intersect at the point (60,48), so the wires cross 48 feet above the ground. ***B***



25. Same approach as the previous problem. Let the bottom of the 120 foot tower be the coordinate (0,0). The top of the tower is (0,120). The wire from the 80 foot building comes to (0,60). The bottom of the 80 foot building is (100,0). The top of the 80 foot building is (100, 80). The wire from the 120 foot building comes to (100, 40). The first wire contains the points (0,60) and (100,80). The equation of this line is y=.2x + 60. The equation of the second wire, which contains (100, 40) and (0,120) is y= -.8x+120.

These lines intersect at the point (60,72), so the wires cross 72 feet above the ground. ***D***



26. Since we do not know how many students are in all three classes, we define the number of students in all three classes as x. We must put each value in terms of x as shown in the diagram. After adding up each of the seven values, we find that x + 69=86. Therefore, there are 17 people in all three classes. ***A***

27. While the pool is filling, we must first find out how long it would take to fill. This is done by solving for x in the equation $\frac{x}{12}+\frac{x}{8}=1$. X=4.8 After two hours, $\frac{2}{4.8}=\frac{5}{12}$ of the pool is full. That means there is $\frac{7}{12}$ of the pool that need to still be filled. After the drain is turned on, we solve the equation $\frac{x}{12}+\frac{x}{8}-\frac{x}{6}=\frac{7}{12}$ to find that there are 14 hours until the pool is full. After 9 hours, $\left(\frac{9}{14}\right)\left(\frac{7}{12}\right)=\left(\frac{3}{8}\right)$ of the remaining pool that have been filled up and $\frac{7}{12}-\frac{3}{8}=\frac{5}{24}$ of the pool that still needs to be filled up. After turning off the drain, we solve $\frac{x}{12}+\frac{x}{8}=\frac{5}{24}$ for x to find that x=1. Thus, it takes the 2 hours until the drain is started, the 9 hours that the drain is turned on, and the hour that it takes after the drain is turned off. Thus, it takes 2 + 9 + 1=12 hours for the pool to fill. ***C***

28. If Beth Sue incorrectly copied the constant term, then she copied the linear term correctly. Thus, she knows that the sum of the zeroes is 7. If Mary Beth incorrectly copied the linear term, then she copied the constant term correctly. Thus, she knows that the product of the zeroes is –120. Mrs. Sue wants the students to solve the polynomial $x^2-7x-120=0$, which has 15 and -8 as its zeroes. ***A***

29. There are 8 ways to pick someone in the first seat, 4 ways to pick the second person since it must be someone of a different sex. Then it follows that there are 3 choices in the 3rd and 4th seats and 2 choices in the 5th and 6th seats. So it looks like $\underline{8}\times\underline{4}\times\underline{3}\times\underline{3}\times\underline{2}\times\underline{2}\times\underline{1}\times\underline{1}$=1152. ***B***

30. We first find the number of ways to sit the men , which is (4-1)!=6. Then, we simply must place the women in any order, as if in a line. Knowing this, there are 4!=24 ways to sit the women. Therefore, 24(6)=144. ***D***