1) $\vec{c}\cdot\vec{d} = 7\cdot 4+(-6)\cdot 3+2\cdot(-6) = -2$

A

2)

$$\frac{|3a+4\cdot 6-10|}{\sqrt{3^2+4^2}} = 7$$

$3a = 21 \rightarrow a = 7$

C

3) Direction components are the coefficients of *t*, and the point components are added on:

$l(t) = (4t-7)i+(t-4)j+(11t)k$

B

4)

$$\begin{vmatrix} 7 & 9 & -1 \\ 3 & -2 & 6 \\ 6 & c & -5 \end{vmatrix} = 0$$

$7(10-6c)-3(-45+c)+6(54-2) = 0$

$517 = 45c \rightarrow [c] = 11$

B

5)

$$\begin{bmatrix} 2 & 4 & 0 & -2 \\ 0 & 1 & -1 & 0 \\ 1 & 2 & -1 & -1 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} 2 & 4 & 0 & -2 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

(3 pivots)

C

6) Entries range from 1 to $10^2 = 100$, so average entry is $\frac{100+1}{2} = 50.5$.

$50.5\cdot 10$ entries per row, column, diagonal $= 505$

B

7)

$$\begin{vmatrix} 1-l & 0 \\ 1/2 & 1/2-l \end{vmatrix} = 0$$

$l^2 - 3/2\, l + 1/2 = 0 \rightarrow l = 1/2, 1$

$$\begin{bmatrix} 1/2 & 0 \\ 1/2 & 0 \end{bmatrix} = 0 \rightarrow \vec{v} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 1/2 & -1/2 \end{bmatrix} = 0 \rightarrow \vec{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

C

8)

1) False: No real eigenvalues, so A does not exist
2) True: $\det(LU) = \det(L)\cdot\det(U)$ and $A = LU$
3) False: Counterexample is $A=I$.
4) False: Counterexample is $A=I$.

B

9)

$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is not of the form

$$\begin{bmatrix} \cos q & -\sin q \\ \sin q & \cos q \end{bmatrix}$$

C

10)

$$\frac{1}{\det(A)}\cdot\det(\text{signed minors of } A^T) =$$

$$\frac{1}{-1}\begin{bmatrix} -1 & 1 & 0 \\ 2 & -2 & -1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 2 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

E

11)

$$\cos q = \frac{\vec{p}\cdot\vec{q}}{\|\vec{p}\|\|\vec{q}\|} = \frac{23}{50}$$

$$|\cos(2q)| = 2\cos^2 q - 1 = \frac{721}{1250}$$

$\sqrt{1250-721} = 23$

B

12) Defintion:

Characteristic polynomial

D

13) $\det(A) = -7$

$\det(AB) = \det(A)\det(B)$

$63 = -7\det(B) \rightarrow \det(B) = -9$

A

14)

$\dim(\square(A^T)) = \#\text{ columns} - \text{rank}$

$= 3-2 = 1$

B

15)

$$\begin{vmatrix} i & j & k \\ 4-1 & 7-2 & 8-3 \\ 3-1 & 2-2 & 6-3 \end{vmatrix} = 15i - j - 10k$$

$$\frac{\sqrt{15^2+1^2+10^2}}{2} \approx 9.0\mathbf{2}8$$

A

16) Trace equal to summation plus $T_{\frac{n+1}{2},\frac{n+1}{2}}$ which is also equal to the sum of the eigenvalues $= \dfrac{n(n+1)}{2}$.

$$\frac{n(n+1)}{2}-\frac{n^2}{2}=\frac{n}{2}$$

B

17) $\vec{x}=\left(A^TA\right)^{-1}A^T\vec{b}$

$$\left(\begin{bmatrix}1&1&1\\0&2&1\end{bmatrix}\begin{bmatrix}1&0\\1&2\\1&1\end{bmatrix}\right)^{-1}\begin{bmatrix}1&1&1\\0&2&1\end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix}$$

$$=\begin{bmatrix}2/3\\0\end{bmatrix}$$

B

18)

$$e^K=1+K+\frac{K^2}{2}+\ldots$$

$$=\begin{bmatrix}1&0\\0&1\end{bmatrix}+\begin{bmatrix}0&1\\0&0\end{bmatrix}+\begin{bmatrix}0&0\\0&0\end{bmatrix}+\ldots$$

$$=\begin{bmatrix}1&1\\0&1\end{bmatrix}\rightarrow 1+1+1=3$$

B

19)

$$A=\begin{bmatrix}13&x\\24-x&13\end{bmatrix}\rightarrow x+24-x=24$$

B

20)

$$\begin{bmatrix}3/2&-1/2\\-1/2&3/2\end{bmatrix}$$

has the same eigenvalues $(2 \text{ and } 1)$

C

21) $N=SD^{10}S^{-1}$, where D is the eigenvalue diagonal matrix of M and S the eigenvector matrix:

$S=\begin{bmatrix}1&1\\0&-1\end{bmatrix}$ and $D=\begin{bmatrix}2&0\\0&1\end{bmatrix}$

$$N=\begin{bmatrix}2^{10}&2^{10}-1\\0&1\end{bmatrix}$$

$$2^{10}-\left(2^{10}-1\right)=1$$

C

22)

$$\frac{\vec{u}\cdot\vec{v}}{\left\|\vec{u}\right\|^2}\vec{u}=\frac{7}{49}\vec{u}=\frac{1}{7}\vec{u}$$

E

23)

$$Z_{re}=\begin{bmatrix}1&0&1&3&4\\0&1&3&2&1\\0&0&0&1&1\\0&0&0&0&0\end{bmatrix}$$

$$Z_{rre}=\begin{bmatrix}1&0&1&0&1\\0&1&3&0&-1\\0&0&0&1&1\\0&0&0&0&0\end{bmatrix}$$

$Z_{re}-Z_{rre}$ has 4 nonzero elements

D

24)

$$A=\begin{bmatrix}1&0&0&0\\1&1&1&1\\1&2&4&8\\1&3&9&27\end{bmatrix}$$

1+1+4+27=33

C

25)

I) True: rows are linearly independent

II) True: each element in A^n decreases for increasing n

III) True: Real- $A=A^H$
Orthogonal- $A=A^{-1}=A^T$

D

26)

$$D^H=\begin{bmatrix}3&1+6i\\4-3i&-2\end{bmatrix}$$

$$3(-2)(1+6i)(4-3i)=$$

$$-6(4+24i-3i+18)=$$

$$-132-126i\rightarrow -126+132=6$$

C

27)

$(AB)^{-1}=B^{-1}A^{-1}$, so product $=$

$2I\cdot I\cdots 2I\cdot I=(2I)^{1001}=$

$$\begin{bmatrix}2^{1001}&0\\0&2^{1001}\end{bmatrix}$$

$$\log_2 2^{1001}=1001$$

B

28)

$$\vec{v_1'}=\frac{(1,2,2)}{\left\|(1,2,2)\right\|}=\tfrac{1}{3}(1,2,2)$$

$$\vec{v_2'}=\frac{\vec{v_2}-\left(\vec{v_2}^T\vec{v_1'}\right)\vec{v_1'}}{\left\|\vec{v_2}-\left(\vec{v_2}^T\vec{v_1'}\right)\vec{v_1'}\right\|}=\tfrac{1}{\sqrt{2}}(0,1,-1)$$

C

29)

$P=\begin{bmatrix}.8&.2\\.75&.25\end{bmatrix}$ and $\vec{p}=\begin{bmatrix}p_c&p_t\end{bmatrix}$

$$\vec{p}P=\vec{p}\rightarrow .8p_c+.75p_t=p_c$$

$$p_t+p_c=1\rightarrow p_t=4/19$$

A

30)

$$\begin{vmatrix}i&j&k\\9&4&0\\7&3&0\end{vmatrix}=-k$$

(1 unit into the page)

B