

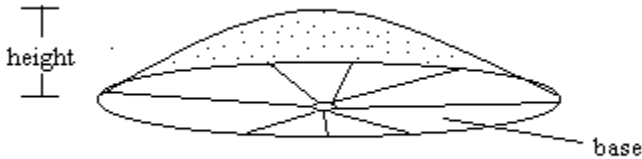
# Analytic Geometry

## FAMAT State convention 2001

**NOTA: None Of These Answers**

1. If  $f(x) = \frac{x}{3-x}$  and  $g(x)$  is the inverse of  $f(x)$ , find  $g(-2) + g(2)$ .  
a) 0      b) 8      c) -4      d) 4      e) NOTA
2. Find the area of the convex pentagon with vertices  $(1,3), (-5,0), (-1,-2), (-5,4), (-3,4)$ .  
a) 20      b) 25      c) 25.5      d) 26      e) NOTA
3. If the distance from the focus to the directrix of a parabola is 3, what is the length of its latus rectum?  
a) 6      b) 3      c) 4      d)  $\frac{3}{2}$       e) NOTA
4. Let E be the center of the ellipse with equation  $\frac{x^2}{4} + \frac{y^2}{3} = 1$ . Let R and T be the endpoints of a latus rectum of the same ellipse. Find the perimeter of RTE.  
a) 4      b)  $\frac{\sqrt{73}+3}{4}$       c)  $3 + \sqrt{13}$       d)  $\sqrt{2} + 5$       e) NOTA
5. A hyperbola has eccentricity  $e$  and a latus rectum of length  $l$ . If  $e = l = 4$ , find the length of that hyperbola's conjugate axis.  
a)  $\frac{4\sqrt{15}}{15}$       b)  $\frac{4}{15}$       c)  $\frac{2\sqrt{15}}{15}$       d)  $\frac{2}{15}$       e) NOTA
6. Find the volume of a parallelepiped whose 3 determining edges are  $\langle 0,3,0 \rangle, \langle 1,0,0 \rangle, \langle 1,1,1 \rangle$ . (*Hint: the volume of a parallelepiped with determining edges  $\hat{x}, \hat{y}, \hat{z}$  is found by  $V = |\hat{x} \cdot (\hat{y} \times \hat{z})|$ )  
a)  $\sqrt{5}$       b)  $3\sqrt{3}$       c) 6      d) 3      e) NOTA*

7. A spherical orange with volume  $\frac{500p}{3} \text{ cm}^3$  has its top cut off as shown. If the height of the cut off piece of orange is 1 cm, what is the area of the base of the cut off piece, in  $\text{cm}^2$ ?



- a)  $25p$       b)  $3p$       c)  $16p$       d)  $9p$       e) NOTA

8. The plane formed by the points  $(1,0,0)$ ,  $(0,1,0)$ , and  $(0,0,1)$  is of form  $Ax + By + Cz + D = 0$ . Find  $A + B + C + D$  if  $\text{gcd}(A, B, C, D) = 1$  and  $A, B, C, D$  are integers..

- a) 1      b) 2      c) 3      d) 4      e) NOTA

9. Find the distance (to the nearest tenth) between the 2 points with spherical coordinates  $\left(4, \frac{p}{4}, \frac{p}{2}\right)$  and  $\left(3, \frac{p}{6}, \frac{p}{3}\right)$ . (All angles are in radians)

- a) 2.9      b) 3.0      c) 3.1      d) 4.2      e) NOTA

10. What is the area of the circle given by the equation  $r = 3 \cos q + 4 \sin q$ ?

- a)  $25p$       b)  $\frac{25p}{4}$       c)  $16p$       d)  $9p$       e) NOTA

11. Which term best describes curves with the following equations:  $(x^2 + y^2 - ax)^2 = a^2(x^2 + y^2)$ ?

- a) cardioid      b) ellipse      c) lemniscate      d) parabola      e) NOTA

12. Find the foci of the conic section with equation  $9x^2 + 25y^2 + 36x - 150y + 36 = 0$ .

- a)  $(-2, 3 \pm 4)$       b)  $(2 \pm 4, -3)$       c)  $(2, -3 \pm 4)$   
 d)  $(-2 \pm 4, 3)$       e) NOTA

13. Find the smallest angle  $q$  (to the nearest tenth of a degree) between the lines  $x + 2y + 3 = 0$  and  $4x + 5y + 6 = 0$ .
- a) 12.6      b) 13.0      c) 11.3      d) 12.9      e) NOTA
14. Parabola  $x = Ay^2 + Cy + M$  is defined by the points  $(3,2)$ ,  $(-1,-1)$ , and  $(0,-3)$ . Find  $A + C + M$ .
- a)  $\frac{14}{15}$       b) 0      c)  $\frac{26}{15}$       d)  $\frac{8}{15}$       e) NOTA
15. Find the shortest distance between the circle  $x^2 + y^2 + 4x - 12y + 31 = 0$  and the point  $(1,2)$ .
- a)  $\sqrt{73} - 3$       b) 2      c)  $\sqrt{73} - 2$       d) 5      e) NOTA
16. Segment  $P_1P_2$  has length  $L$  and endpoints  $P_1 = (2,7)$  and  $P_2 = (8,2)$ . Find a point on  $P_1P_2$  that is  $\frac{1}{3}L$  away from  $P_1$ .
- a)  $\left(6, \frac{11}{3}\right)$       b)  $(4,5)$       c)  $\left(4, \frac{16}{3}\right)$       d)  $\left(\frac{11}{3}, \frac{16}{3}\right)$       e) NOTA
17. Find the acute angle  $q$ , in radians, that the equation  $x^2 + 2xy + y^2 + 4x - 5y + 6 = 0$  must be rotated through to eliminate the  $xy$  term.
- a)  $\frac{p}{3}$       b)  $\frac{p}{4}$       c)  $\frac{p}{5}$       d)  $\frac{p}{6}$       e) NOTA
18. The line  $Ax + By + C = 0$  is defined to be the negative-slope asymptote of the hyperbola with equation  $x^2 - 4y^2 - 8x - 16y - 16 = 0$ . Find  $A + B + C$  where  $A, B,$  &  $C$  are integers and  $\gcd(A, B, C) = 1$ .
- a) 0      b) 2      c) -3      d) 3      e) NOTA
19. Find the shortest distance between the parallel lines with equations  $5x - 12y + 33 = 0$  and  $5x - 12y - 6 = 0$ .
- a) 39      b) 3      c)  $\frac{27}{5}$       d)  $\frac{27}{13}$       e) NOTA
20. Exactly how many points of intersection are there between the conic sections with equations  $9y^2 - 4x^2 + 36 = 0$  and  $9x^2 + 16y^2 - 18x - 135 = 0$ ?
- a) 1      b) 2      c) 3      d) 4      e) NOTA

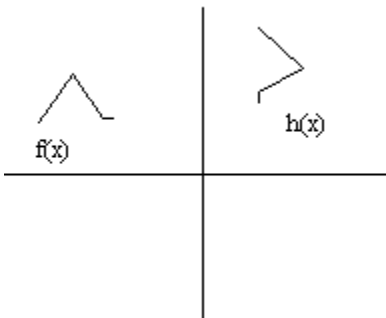
21. The sum of the distances from point  $P$  to the points  $(-2,0)$  and  $(4,0)$  is 10. Find the equation for the locus of all such points.

- a)  $\frac{x^2}{25} + \frac{y^2}{16} = 1$       b)  $\frac{(x-1)^2}{25} + \frac{y^2}{9} = 1$       c)  $\frac{(x-1)^2}{25} + \frac{y^2}{16} = 1$   
 d)  $\frac{x^2}{25} + \frac{y^2}{9} = 1$       e) NOTA

22. The positive difference of the distances from point  $P$  to the points  $(-2,0)$  and  $(4,0)$  is 4. Find the equation for the locus of all such points.

- a)  $\frac{(x-1)^2}{4} - \frac{y^2}{5} = 1$       b)  $\frac{(x-1)^2}{16} - \frac{y^2}{25} = 1$       c)  $\frac{x^2}{16} - \frac{(y-1)^2}{25} = 1$   
 d)  $\frac{x^2}{16} - \frac{y^2}{9} = 1$       e) NOTA

23. If  $f(g(x)) = x$ , then  $h(x)$  is approximately equivalent to which of the following?



- a)  $-g(f(x))$       b)  $-g(x)$       c)  $f(-x)$   
 d)  $g(-x)$       e) NOTA

24. The circle with equation  $x^2 + y^2 + Ax + By + C = 0$  is defined by the points  $(-4,-1)$ ,  $(1,-1)$  and  $(-1,4)$ . Find  $A + B + C$ .

- a) 1.2      b) 0      c) -4      d) -5.6      e) NOTA

25. A certain circle has a chord of length 12 that is tangent to a smaller, concentric circle. Find the area between the two circles.

- a)  $6p$       b)  $9p$       c)  $24p$       d)  $36p$       e) NOTA

26. A triangle with sides of length 4, 4, and 6 is inscribed in a circle. Find the area of that circle.

- a)  $\frac{64p}{7}$       b)  $\frac{16p}{7}$       c)  $\frac{9p}{7}$       d)  $9p$       e) NOTA

27. There is a triangle with sides of length  $M$ ,  $D$ , and  $S$ . If  $D + S = 2M$  and  $DS = M^2$ . Find the angle  $q$  (in radians) opposite the side of length  $M$  if  $0 < q < \frac{p}{2}$ .

- a)  $\frac{p}{6}$       b)  $\frac{p}{4}$       c)  $\frac{p}{3}$       d)  $\frac{5p}{12}$       e) NOTA

28. A man standing atop a watchtower sees a ship at an angle of depression of  $15^\circ$ . He looks at the ship again minutes later and sees it at an angle of depression of  $30^\circ$ . If the ship traveled 600 feet toward the watchtower in that time, how high above sea level is the man, to the nearest hundredth of a foot?

- a) 300.00      b) 519.62      c) 1119.62      d) 821.81      e) NOTA

29. A CD spins with an angular velocity of 4500 rotations/min. What is the linear velocity, in  $\frac{\text{ft}}{\text{sec}}$ , of a point 2 inches from the center of the CD?

- a)  $300p$       b)  $\frac{75}{4p}$       c)  $\frac{25}{16p}$       d)  $25p$       e) NOTA

30. What is the slope of the line that passes through the points of intersection of the graphs  $y = \log_4 x$  and  $y = 4^{x-1} - 1$ ?

- a)  $\frac{1}{4}$       b)  $\frac{1}{2}$       c) 1      d) 2      e) NOTA

