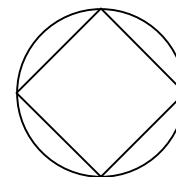


Mu Topic Test - Circumference, Perimeter, Area, and Volume  
FAMAT State Convention 2001

For all questions, answer "E) NOTA" means none of the above answers is correct.  
The figures in this test are not drawn to scale.

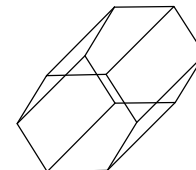
- 1) The figure below contains a square inscribed a circle with a circumference of  $\pi$ . Find the perimeter of the square.

- A)  $\frac{1}{2}$       B)  $\frac{\sqrt{2}}{2}$       C)  $2\sqrt{2}$       D)  $\pi^2$       E) NOTA



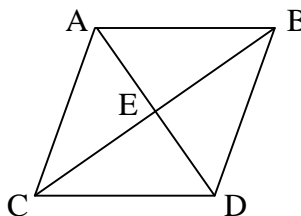
- 2) The figure below is a regular right prism with a height of 12cm and a hexagonal base of side length 5cm. Find the volume of the prism.

- A)  $75\sqrt{3}$       B)  $180\sqrt{3}$       C)  $300\sqrt{3}$       D)  $450\sqrt{3}$       E) NOTA



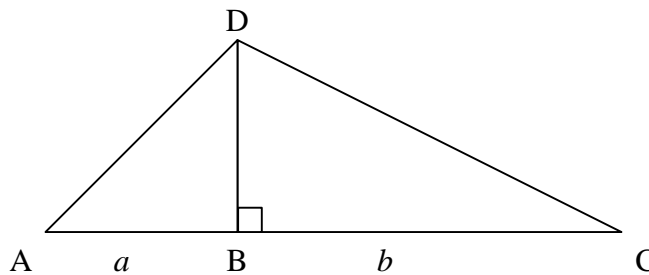
- 3) In the rhombus below,  $\overline{AD}$  and  $\overline{CB}$  are diagonals. If  $AD = 6$  and  $CB = 8$ , find the perimeter of  $ABDC$ .

- A) 14  
B) 20  
C) 28  
D) 48  
E) NOTA



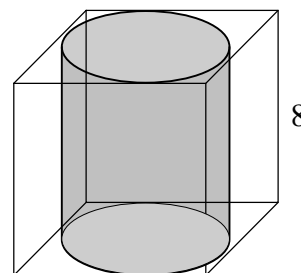
- 4) Find the sum of the perimeters of  $\triangle BAD$  and  $\triangle CDB$  in the figure below in terms of "a" and "b" assuming  $m\angle ADC = m\angle DBC = 90^\circ$ .

- A)  $a + b + 2\sqrt{ab} + \sqrt{a^2 + ab} + \sqrt{ab + b^2}$   
B)  $\sqrt{a} + \sqrt{b} + 2\sqrt{ab} + \sqrt{a^2 + ab} + \sqrt{ab + b^2}$   
C)  $a + b + \sqrt{ab} + \sqrt{a^2 + ab} + \sqrt{ab + b^2}$   
D)  $a^2 + b^2 + 2\sqrt{ab} + \sqrt{a^2 + ab} + \sqrt{ab + b^2}$   
E) NOTA



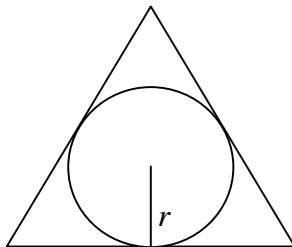
- 5) A container full of water is in the shape of a cube with side length 8 as in the figure below. The largest cylinder that will fit in the cube is pushed down into the water causing an overflow out of the cubic container. Find the volume of water remaining in the cube after the cylinder is completely inside the cube.

- A)  $64\pi$   
B)  $512 - 64\pi$   
C)  $128\pi$   
D)  $512 - 128\pi$   
E) NOTA



**The following figure applies to questions 6 and 7.**

The figure below has a circle of radius “ $r$ ” inscribed in an equilateral triangle.



6) Find the ratio of the circumference of the circle to the perimeter of the triangle.

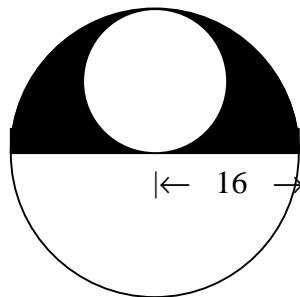
- A)  $\frac{p\sqrt{3}}{9}$     B)  $\frac{2p\sqrt{3}}{9r}$     C)  $\frac{2p\sqrt{3}}{9}$     D)  $\frac{p\sqrt{3}}{r}$     E) NOTA

7) Find the ratio of the area of the circle to the area of the triangle.

- A)  $\frac{p\sqrt{3}}{9}$     B)  $\frac{2p\sqrt{3}}{9r}$     C)  $\frac{2p\sqrt{3}}{9}$     D)  $\frac{p\sqrt{3}}{r}$     E) NOTA

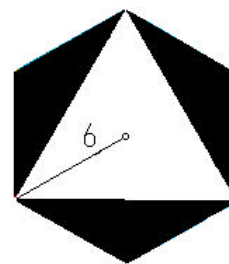
8) In the figure below, the smaller circle is tangent to the larger circle and is tangent to the diameter at its midpoint. The radius of the larger circle is 16 units as indicated. What is the area of the shaded region in the figure below?

- A)  $18\pi$   
 B)  $36\pi$   
 C)  $64\pi$   
 D)  $128\pi$   
 E) NOTA



9) A triangle is within a regular hexagon such that they have three common vertices, as in the figure. Find the area of the shaded region given that the radius of a circumscribed circle would be 6.

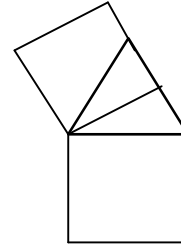
- A)  $27\sqrt{3}$   
 B)  $54\sqrt{3}$   
 C)  $81\sqrt{3}$   
 D)  $108\sqrt{3}$   
 E) NOTA



10) One diagonal of a square serves as the shorter base of a trapezoid. A line through one of the vertices of the square contains the other base. The legs of the trapezoid are extensions of two sides of the square. If the area of the square is 100, what is the area of the trapezoid?

- A) 75    B) 150    C) 225    D) 300    E) NOTA

- 11) The figure below shows an equilateral triangle with area  $4\sqrt{3}$  and two squares. An altitude of the triangle is a side of one of the squares. The other square shares a side with the triangle. Find the ratio of the area of the smaller square to the area of the larger square.



- A) 1:1
- B) 2:3
- C) 3:4
- D) 4:5
- E) NOTA

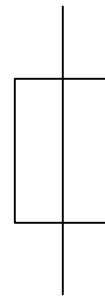
- 12) Find the area of a triangle with sides of length 5, 7, and 8.

- A)  $\frac{20}{3}$
- B)  $10\sqrt{3}$
- C)  $5\sqrt{6}$
- D)  $\sqrt{30}$
- E) NOTA

**For #13 and #14 the figures are rotated about the axis (shown as a dotted line). Find the volume of the figure formed by this rotation.**

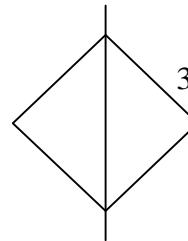
- 13) A rectangle of height 6 and width 4 is rotated to form a cylinder.

- A)  $12\pi$
- B)  $24\pi$
- C)  $72\pi$
- D)  $76\pi$
- E) NOTA



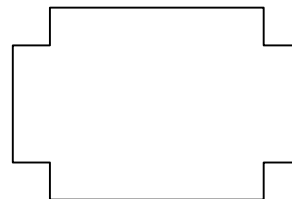
- 14) A square of side length 3 as shown is rotated to form a bicone.

- A)  $p\sqrt{2}$
- B)  $3p$
- C)  $\frac{9p\sqrt{2}}{4}$
- D)  $\frac{9p\sqrt{2}}{2}$
- E) NOTA



- 15) A piece of 8.5 by 11 inch paper is cut as shown below so as to form a box when folded. If squares of side length “ $x$ ” are cut out, find the volume of the box if  $x = 2$  inches.

- A) 63
- B) 63.5
- C) 95
- D) 117
- E) NOTA



16) A pendulum four feet long “sweeps” an arc of  $11.25^\circ$  on either side of its center position. Find the area of the sector “swept” out by this pendulum.

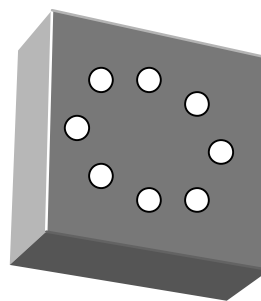
- A)  $\frac{p}{6}$       B)  $\frac{p}{4}$       C)  $\frac{p}{2}$       D)  $p$       E) NOTA

17) James wishes to approximate the volume of a vase by filling it with a known number of marbles each of a known volume. In order to find the best (most accurate) approximation of this volume, what value must the radius of each marble approach?

- A) 0      B)  $\frac{1}{p}$       C)  $p$       D)  $\infty$       E) NOTA

18) Hilda arranges eight pegs in a board as shown below. If the distance between each peg is 6 centimeters, what is the minimum amount of string that can be used in order to touch each and every peg (assume the area/volume of each peg is negligible)?

- A) 42  
B) 48  
C) 54  
D) 720  
E) NOTA

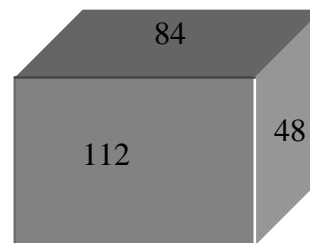


19) A segment (of a circle) is created when two radii of a circle form the legs of an isosceles right triangle. If the circumference of this circle is  $4\pi$ , find the area of the segment.

- A)  $\pi-2$       B) 2      C)  $\pi$       D)  $3\pi$       E) NOTA

20) You know the surface area of three faces of a box (rectangular prism) to be 112, 84, and 48. These represent the front, top, and side of the box respectively as shown below. Find the volume of this box.

- A) 76.72  
B) 244  
C) 672  
D) Can't be determined  
E) NOTA



21) At a track and field competition two runners run around a track formed by two “straights” and two semi-circles for the curved ends. The length of each straight is 500 feet and the length of each inside curve is 160 feet. These given dimensions represent the inside measurements of the track. The track is a uniform 15 feet wide. If runner A runs along the inside track wall running the minimum distance each time around and runner B runs along the outside track wall running the maximum distance each time, how much further does runner B run each lap?

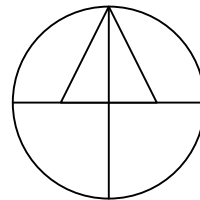
- A) 0      B)  $15\pi$       C)  $30\pi$       D)  $60\pi$       E) NOTA

22) Find the area of a rhombus with diagonals of length 10 and 24.

- A) 34      B) 54      C) 120      D) 169      E) NOTA

23) Two vertices of an equilateral triangle lie on a diameter of a circle and the third vertex lies on the circle whose area is  $64\pi$  as shown below. The dotted line (a diameter) is the perpendicular bisector of the solid diameter and intersects the triangle at the vertex tangent to the circle. Find the area of the triangle.

- A)  $\frac{8\sqrt{3}}{3}$       C)  $\frac{64\sqrt{3}}{3}$       E) NOTA  
 B)  $\frac{16\sqrt{3}}{3}$       D)  $\frac{64p}{3}$



24) A triangle has side lengths of 4, 5, and 7. What is the perimeter of the largest triangle that is similar to the original and has a side of length 15.

- A)  $\frac{240}{7}$       B) 45      C) 48      D) 60      E) NOTA

25) A circle of radius 4 has a circumference equal to the perimeter of a square. Find the length of a diagonal of this square.

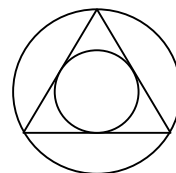
- A)  $2p$       B)  $(2p)^2$       C)  $p(2^{2/3})$       D)  $2p\sqrt{2}$       E) NOTA

26) A regular hexagon of side length “ $x$ ” is split into three congruent pieces. Find the area of one of these sections in terms of “ $x$ ”.

- A)  $\frac{3x^2}{2}$       C)  $\frac{x^2\sqrt{3}}{2}$       E) NOTA  
 B)  $\frac{x^2\sqrt{3}}{6}$       D) A hexagon cannot be divided into three equal pieces.

27) A circle of area  $36\pi$  is inscribed in an equilateral triangle that is inscribed in a larger circle as shown below. Find the area of the larger circle.

- A)  $2\pi$       C)  $81\pi$       E) NOTA  
 B)  $64\pi$       D)  $144\pi$



28) A widget obeys Euler’s Formula. If a particular widget has 26 edges and 18 vertices and each face has an area of 3, find the surface area of this widget.

- A) 10      B) 15      C) 30      D) 45      E) NOTA

29) The midpoints of the sides of an equilateral triangle are connected to form a new triangle. Find the ratio of the area of the smaller triangle to the original triangle.

- A) 1:6      B) 1:4      C) 1:2      D) 2:1      E) NOTA

30) Find the arc length “swept” by a 2 inch hour hand during a 6 hour 20 minute interval.

- A)  $\frac{18p}{9}$       B)  $\frac{19p}{9}$       C)  $\frac{20p}{9}$       D)  $\pi$       E) NOTA