Limits and Derivatives – Calculus Division
FAMAT State Convention 2001

For all questions, answer E. NOTA means none of the above answers are correct.
Note: For all questions, the abbreviation DNE denotes 'Does Not Exist.'

1. Solve: $\lim_{x \to 2001} \frac{x+2000}{x-2000} = ?$

A. 1 B. 2001
C. 4001 D. DNE E. NOTA

2. David's annual MAT poem (a limerick!):

I examine several functions: *p*, *f*, and *q*,
And I notice $p \le f \le q$ on the domain interval (t,u).
If the limit as *x* approaches *c* of both *p* and *q* is three,
And if $t \le c \le u$, it's plain to see,
That $\lim_{x \to c} f(x) = 3$ too.

Which of the following theorems has been applied in this limerick?

A. Squeeze Theorem B. Rolle's Theorem
C. Fundamental Theorem of Calculus
D. First Derivative Theorem E. NOTA

3. If $f(x) = 6x^2 - 9x + 2$ and $\int f(x)dx = F(x)$ then $F'(-1) = ?$

A. –21 B. –1
C. 3 D. 17 E. NOTA

4. Solve: $\lim_{x \to \infty} \frac{4x^2 - 8x + 5}{3x^3 + 4x^2} = ?$

A. 0 B. $\frac{4}{3}$
C. $\frac{8}{9}$ D. 8 E. NOTA

5. Which of the following describes the graph and concavity, respectively, of $g(x) = \frac{1}{2}x^4 - x^3 - 10x^2 + 3$ over the domain interval $(1,2)$?

A. Increasing, Up B. Increasing, Down
C. Decreasing, Up D. Decreasing, Down
E. NOTA

6. If a particle moves with position (along a line) given by $d(t) = 6t^3 - 14t^2$ for $t \ge 0$, determine its speed at $t = 1$.

A. –8 B. 8
C. –10 D. 10 E. NOTA

7. $Z(x)$ is continuous on [a,b], differentiable on (a,b), and $Z(a) = Z(b)$. Which theorem states that there exists some number *c*, $b > c > a$, such that $Z'(c) = 0$?

A. Squeeze Theorem B. Rolle's Theorem
C. Fundamental Theorem of Calculus
D. First Derivative Theorem E. NOTA

8. Which of the following ordered pairs (A, B) makes the following statement true:

$$\lim_{x\to\infty} \frac{3x^5 + 2x + \frac{1}{x}}{x^A} = B\,?$$

A. (5,0) B. (6,1)
C. (5,3) D. $(10, \infty)$ E. NOTA

9. An ellipse whose top half has equation $f(x) = y$ has center $C(0,0)$ with two vertices at $V(0,3)$ and $V(2,0)$. What is the value of $\frac{dy}{dx}$ at the point $(1, \frac{3\sqrt{3}}{2})$?

A. $\frac{\sqrt{3}}{2}$ B. $\frac{-\sqrt{3}}{2}$

C. $\frac{\sqrt{3}}{3}$ D. $\frac{-\sqrt{3}}{3}$ E. NOTA

10. Given $x_0 = 3$ and $h(x) = 10 - x^2$, use one iteration of Newton's method to find x_1.

A. $2\frac{5}{6}$ B. $3\frac{1}{6}$
C. 9 D. $9\frac{1}{6}$ E. NOTA

11. A completely healable wound heals such that at t days after the wound appears, the area (in square units) that the wound covers is $W(t)$. Which of the following functions could be $W(t)$?

A. $W(t) = 2t$ B. $W(t) = \cos t$
C. $W(t) = \frac{3t+2}{t}$ D. $W(t) = \sin\left(\frac{p}{2t}\right)$ E. NOTA

12. Given $M(x) = f'(x)$, $A(x) = \frac{f(x)}{x}$, and $x \neq 0$, where do the graphs of $M(x)$ and $A(x)$ intersect (assume $f(x)$ is everywhere continuous and differentiable)?

A. At a critical point of $M(x)$
B. At a critical point of $A(x)$
C. At a critical point of $f(x)$
D. At $f(x) = 0$ E. NOTA

13. What is the limit of the sequence $2, 3, 3\frac{1}{2}, \ldots, 4 - \frac{4}{2^n}, \ldots$ as n approaches infinity?

A. 0 B. 2
C. 4 D. diverges E. NOTA

14. For $w(x) = \sqrt{x} - 9$, use a differential to approximate $w(100 + 2)$.

A. 0.2 B. 1.0
C. 1.2 D. 10.1 E. NOTA

15. For $f(x) = \cos(x) + \sin(x)$, determine the value of $\sum_{i=1}^{2001} f^i\left(\frac{p}{2}\right)$, where $f^i(x)$ denotes the ith derivative of f.

A. 0 B. 1
C. 249 D. 251 E. NOTA

16. Let $g(x) = x^2 - 9x + 12$ and $h(x) = 4x^3 - 5x^2$. Determine $(g \circ h)'(2)$.

A. 15 B. 48
C. 140 D. 420 E. NOTA

17. A square sheet of metal has side length 10 inches. Squares of length *x* inches are cut from each corner, and the resulting 'flaps' are folded up to form an open-topped box. What value of *x* maximizes the volume of the box?

A. $\frac{5}{3}$ B. $\frac{10}{3}$

C. 5 D. 10 E. NOTA

18. Find the minimum distance between the graph of $y = x^2 - 2x$ and the point (1,0).

A. –2 B. –1
C. 1 D. 2 E. NOTA

19. If $(x) = Arc\cos(x)$, find $g'\left(\frac{1}{2}\right)$.

A. $\sin\left(\frac{1}{2}\right)$ B. $\frac{2\sqrt{3}}{3}$

C. $\frac{4}{5}$ D. $\frac{p}{3}$ E. NOTA

20. The point where $x = -3$ on the graph of $f(x) = \frac{x^3 + 2x^2 - 24x}{x - 4}$ represents which of the following?

A. Relative Minimum B. Relative Maximum
C. Inflection Point D. Root E. NOTA

21. Which of the following functions is both differentiable and continuous at $x = 5$?

A. $f(x) = \frac{\sqrt{5-x}}{\sqrt{5+x}}$ B. $f(x) = \frac{x^3 + x^2 - 46x + 80}{x^2 - 2x - 15}$

C. $f(x) = |5x - 25|$ D. $f(x) = \frac{\tan x + \sin x}{\tan x - \sin x}$

E. NOTA

22. Solve: $\lim_{x \to 0^+} \frac{x^{\frac{5}{2}} - 2x + 1}{6x^2 + 3\sqrt{x} + 2} = ?$

A. $\frac{-1}{2}$ B. 0

C. $\frac{1}{2}$ D. DNE E. NOTA

23. The Mean Value Theorem states that given $f(x) = x^3 - 2x^2 + 3x - 4$, there exists some *c* on the interval (2,4) such that $f'(c)$ equals which of the following?

A. 19 B. 38
C. 43 D. 99 E. NOTA

24. Let $p(x) = 10^x$. Which of the following is true for $p^n(x)$, the graph's nth derivative?

A. $p^n(x) = n!\,p(x)$ B. $p^n(x) = n!\ln(10)\,p(x)$

C. $p^n(x) = (\ln(10))^n\,p(x)$ D. $p^n(x) = p(x)$

E. NOTA

25. The circle defined by $(x-3)^2+(y+7)^2=25$ is tangent to the line $4x-3y=-2B$ at the point (A,B). Find the sum $A+B$.

A. –5 B. –1
C. 1 D. 5 E. NOTA

26. Solve: $\lim_{x\to 1}\dfrac{e^{1-x}+3\cos(\pi x)+2\sqrt{x}}{3x^3-\dfrac{3}{x}-2x+\dfrac{2}{x^2}}=?$

A. –1 B. –0.5
C. 0 D. DNE E. NOTA

27. A rectangle of area 48 square units is drawn to minimize the distance between the midpoints of any two adjacent sides. In units, what is the perimeter of this rectangle?

A. $4\sqrt{3}$ B. $8\sqrt{3}$ C. $16\sqrt{3}$
D. No Minimum Value Exists E. NOTA

28. Let $f(x)=Ax^3+Bx^2+Cx+D$. The graph of f has an inflection point at $x=-1.5$, a relative extrema at $x=1$ and a root at $x=0$. If $A=\dfrac{1}{3}$, find the product *ABCD*.

A. 0 B. –1
C. –2 D. –9 E. NOTA

29. If $p(x)=(1+x)^{\frac{1}{x}}$ and $q(x)=\dfrac{10^x-1}{x}$, determine the value of $\lim_{x\to 0}p(x)q(x)$.

A. $e\ln 10$ B. $10\log e$
C. e^{10} D. $10e$ E. NOTA

30. A spherical ball *B* of radius 3 units begins deflating such that its radius decreases 0.2 units per minute, but it retains its spherical shape. At the same time, a cylinder *C* of radius 4 units and height 3 units begins increasing in height at the rate of 0.3 units per minute (its radius is constant). If *Z* (in minutes) is the first time at which the volume of *B* is decreasing at the same absolute rate as the volume of *C* is increasing, determine $[Z]$, the greatest integer function of *Z*.

A. 2 B. 3 C. 4 D. 27 E. NOTA