C 1. $\frac{2001+2000}{2001-2000}=\frac{4001}{1}=4001.$

A 2. The Squeeze Theorem has been applied.

D 3. $F'(-1)=f(-1)=6(-1)^2-9(-1)+2=17$.

A 4. Since highest exponent in denominator (3) exceeds highest exponent in numerator (2), limit will approach 0.

D 5. $g'(x)=2x^3-3x^2-20x$. Set first derivative to 0: $0=x(2x+5)(x-4)$ so extreme values exist at x = -2.5, 0, and 4. Test a value between 0 and 4 to determine that *g* is decreasing on (0,4) and therefore on (1,2). $g''(x)=6x^2-6x-20$. Set second derivative equal to 0 and use the quadratic formula to find roots: x ˜ -1.39 or 2.39. Test in the interval to find that *g* is concave down on the interval (-1.39, 2.39) and therefore on (1,2).

D 6. $v(t)=d'(t)=18t^2-28t$ and $v(1)=18-28=-10$. The question asked for speed, which is $|v(t)|$ or 10.

B 7. Rolle's Theorem.

C 8. $\lim_{x\to\infty}\frac{3x^6+2x^2+1}{x^{A+1}}=B$. Therefore:

$$B=\begin{cases}0 \text{ if } 6<(A+1)\\ 3 \text{ if } 6=(A+1)\\ \infty \text{ if } 6>(A+1)\end{cases}$$ Only (5,3) satisfies this condition.

B 9. $\frac{x^2}{4}+\frac{y^2}{9}=1$. Differentiate: $\frac{2xdx}{4}+\frac{2ydy}{9}=0$. Plug in the point: $\frac{2(1)dx}{4}+\frac{2\left(\frac{3\sqrt{3}}{2}\right)dy}{9}=0$ and $\frac{dy}{dx}=\frac{-\sqrt{3}}{2}$.

B 10. $x_1=x_0-\frac{h(x_0)}{h'(x_0)}=x_0-\frac{10-x_0^{\,2}}{-2x_0}=3-\frac{10-3^2}{-2(3)}=3\frac{1}{6}$.

D 11. If the wound is completely healable, $\lim_{t\to\infty}W(t)=0$. For choice A, B, and C, respectively, the limit diverges, does not exist, and is 3. Only for choice D is the limit 0.

B 12. We are looking for the point at which $f'(x)=\frac{f(x)}{x}$.

Test choice B: $A'(x)=\frac{f'(x)x-f(x)}{x^2}=0$. Thus,

$\frac{f'(x)x}{x^2}=\frac{f(x)}{x^2}$ Multiply both sides by *x*: $f'(x)=\frac{f(x)}{x}$.
The graphs intersect at $A'(x)=0$.

C 13. $\lim_{n\to\infty}(4-2/n)=4-0=4.$

E 14. $w(x+h)\approx w(x)+w'(x)(h)=\sqrt{x}-9+\frac{h}{2\sqrt{x}}$.

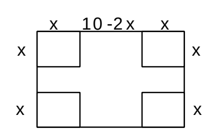
$w(100+2)\approx\sqrt{100}-9+\frac{2}{2\sqrt{100}}=10-9+\frac{1}{10}=1\frac{1}{10}$.

E 15. $f'(x)=-\sin x+\cos x;\ f'(.5p)=-1+0=-1$
$f''(x)=-\cos x-\sin x;\ f''(.5p)=0+-1=-1$
$f'''(x)=\sin x-\cos x;\ f'''(.5p)=1+0=1$
$f^{iv}(x)=\cos x+\sin x;\ f^{iv}(.5p)=0+1=1$

The pattern repeats every fourth derivative, and the sum of the first four derivatives at .5p is 0. Since 2000 is divisible by four, the summation
$=f^{2001}\left(\frac{p}{2}\right)=f'\left(\frac{p}{2}\right)=-1$.

D 16. $(g\circ h)'(2)=g'(h(x))h'(x)$. $g'(x)=2x-9$;
$h'(x)=12x^2-10x.\ h(2)=4(2)^3-5(2)^2=12$.
$h'(2)=12(2)^2-10(2)=28$.
$g'(h(2))=g'(12)=2(12)-9=15.\ \ 15\bullet 28=420.$

A 17. $V(x)=(10-2x)^2(x)=$
$100x-40x^2+4x^3.$
$V'(x)=100-80x+12x^2$
Set derivative equal to 0:
$0=4(3x-5)(x-5)$. So
$x=\frac{5}{3}$ or 5 , but omit x=5 since 2x must be less than 10.
(Check that this is a max with the second derivative.)



C 18. Determine the distance between (1,0) and a point on the graph: $(x, x^2 - 2x)$. Distance= $\sqrt{(x-1)^2 + (x^2 - 2x)^2} = \sqrt{x^4 - 4x^3 + 5x^2 - 2x + 1}$. We can minimize the square of distance: $D' = 4x^3 - 12x^2 + 10x - 2$. Set derivative to 0, and determine that x=1 is a root. In the parabola, if x=1, y=1-2=-1 and the distance from (1,-1) to (1,0) is 1.

B 19. $g'(x) = \frac{1}{\sqrt{1-x^2}}$ so $g'(.5) = \frac{1}{\sqrt{1-(.5)^2}} = \frac{1}{\sqrt{.75}} = \frac{2\sqrt{3}}{3}$.

A 20. $f(x) = \frac{x(x+6)(x-4)}{(x-4)} = x^2 + 6x$. $f'(x) = 2x + 6$ and $f'(-3) = 0$. Since $f''(x) = 2$, all extrema are minima.

D 21. A: $\lim_{x \to 5^+} f(x)$ does not exist. B: There is a hole at $f(5)$ (does not exist). C: The graph is V-shaped with x=5 as the turning point, so no derivative exists. D: The graph is both continuous and differentiable at x=5.

C 22. $\lim_{x \to 0^+} \sqrt{x} = 0$ so we substitute: $\frac{0^5 - 2(0) + 1}{6(0)^2 + 3(0) + 2} = \frac{1}{2}$.

A 23. The MVT states that $f'(c) = \frac{f(4) - f(2)}{4 - 2} = \frac{40 - 2}{2} = 19$.

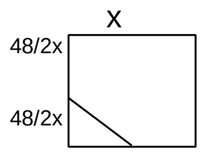
D 24. $p'(x) = \ln(10)10^x$; $p''(x) = \ln(10)\ln(10)10^x$. This pattern will continue such that $p^n(x) = (\ln(10))^n p(x)$.

A 25. The line's slope is $\frac{4}{3}$, which is also dy/dx for the circle. Differentiate the equation of the circle: $2(x-3)dx + 2(y+7)dy = 0 \Rightarrow \frac{3-x}{7+y} = \frac{dy}{dx}$. Plug the point (A,B) into the equation and set it equal to

the line's slope: $\frac{4}{3} = \frac{3-A}{7+B} \Rightarrow 4B + 3A = -19$. Plug the point (A,B) into the line: $4A - 3B = -2B$
$\Rightarrow B = 4A$ .Substitute into the last equation:
$4(4A) + 3A = -19 \Rightarrow A = -1, B = -4$. Sum is –5.

C 26. Limit = $\frac{1-3+2}{3-3-2+2} = \frac{0}{0}$. Use l'Hopital's theorem:
$\lim_{x \to 1} \frac{-e^{1-x} - 3p\sin(px) + \frac{1}{\sqrt{x}}}{9x^2 + \frac{3}{x^2} - 2 - \frac{4}{x^3}}$. Plug in x=1: $\frac{-1-0+1}{9+3-2-4} = \frac{0}{6} = 0$.

C 27.



Distance between midpoints: $\sqrt{\left(\frac{48}{2x}\right)^2 + \left(\frac{x}{2}\right)^2} = \sqrt{\frac{576}{x^2} + \frac{x^2}{4}}$. We can minimize the distance squared: $D' = \frac{-1152}{x^3} + \frac{x}{2}$. Set first derivative to zero and solve to get x= $4\sqrt{3}$ and $\frac{48}{2x} = 4\sqrt{3}$ (a square!). (Use second derivative to verify the value is a min.) Find perimeter: $4 \bullet 4\sqrt{3} = 16\sqrt{3}$.

A 28. Note that if $f(0) = 0$, D (and therefore the product ABCD) must be 0. By the way, $B = \frac{3}{2}$ and $C = -4$.

A 29. $\lim_{x \to 0} p(x)q(x) = \lim_{x \to 0} p(x) \lim_{x \to 0} q(x)$. By definition, $\lim_{x \to 0} p(x) = e$ and $\lim_{x \to 0} q(x) = \ln 10$ so product=$e\ln 10$.

A 30. $Cylinder: V(r,h) = pr^2h \Rightarrow V'(r,h) = 2prhdr + pr^2dh.$
We know r=4, dr=0, and dh=.3:
$V' = 0 + p(16)(.3) = 4.8p$.

$Ball: V(r) = \frac{4}{3}pr^3 \Rightarrow V'(r) = 4pr^2dr$. We know dr=-.2 and r=3-.2t: $V' = -.8p(3 - .2t)^2$. We are looking for the point at which the (absolute value) of the change is V is the same for both figures:
$4.8p = .8p(3-2t)^2 \Rightarrow (3-2t)^2 = 6 \Rightarrow 3 - 2t = \pm\sqrt{6}$.
From here, $t \approx 2.75, t \approx 27.25$ so $[Z] = 2$.

Limits and Derivatives – Answer Key

FAMAT State Convention 2001

1. C
2. A
3. D
4. A
5. D
6. D
7. B
8. C
9. B
10. B
11. D
12. B
13. C
14. E
15. E
16. D
17. A
18. C
19. B
20. A
21. D
22. C
23. A
24. D
25. A
26. C
27. C
28. A
29. A
30. A