Integration Test Solutions

FAMAT 2000

1. B Since even functions are symmetric to the y-axis, the graph between –7 and –1 encloses the same area as between 1 and 7 and has the same amounts above and below the x-axis, so $\int_0^7 f(x)dx = \int_0^1 f(x)dx + \int_1^7 f(x)dx \Rightarrow 1 = 5 + \int_1^7 f(x)dx = -4$.

2. B Area of Rect. 1 = $2 \cdot f(1) = 2 \cdot 1 = 2$; Area of Rect. 2 = $2 \cdot f(2) = 2 \cdot 2 = 4$; and the Ares of Rect. 3 = $2 \cdot f(4) = 2 \cdot 10 = 20$. Adding we get 26.

3. C By the Fund. Thm of Calc., $\int_0^3 R(t)dt$ is the total amount of water that leaks out in the first 3 hours.

4. E The given expression is equiv. to $\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$, where $\Delta x = \frac{1}{n}, x_k = 1 + \frac{k}{n}$ and $f(x_k) = \frac{1}{x_k}$. The given limit is the area under the curve $y = \frac{1}{x}$ from x = 1 to x = 2. Thus, the given limit is equiv. to $\int_1^2 \frac{1}{x}dx$.

5. D $\int_2^4 f'(g(x))g'(x)dx = \int_2^4 f'(u)du = f(u) + c$ or $f(g(x))\Big|_2^4 = f(g(4)) - f(g(2))$.

6. D Graph the 2 functions in the given interval. Choices A thru D represent the areas of the regions enclosed by the graphs from left to right. It is easy to see that choice D is the largest.

7. C $F'(x) = -\frac{d}{dx}\int_1^x \frac{2}{1+t}dt = \frac{-2}{1+x} \Rightarrow F'(1) = \frac{-2}{1+1} = -1.$

8. C Let u = x + c, then du = dx, and $u_1 = 0,\ u_2 = 1 + c \Rightarrow \int_0^{1+c} g(u)du = 10 = -\int_{1+c}^{0} g(u)du = -10.$

9. B The integral must be split at x = 2, to give $\int_0^2 -(x-2)dx + \int_2^6 (x-2)dx = 2 + 8 = 10.$

10. D $\int_1^k \frac{x+1}{x}dx = \int_1^k (1 + \frac{1}{x})dx = (x + \ln|x|)\Big|_1^k = k + \ln k - 1 = k + 1 \Rightarrow \ln k = 2 \Rightarrow k = e^2.$

11. B The average circ. Of all circles with radii btwn 1cm and 3cm is found by $\overline{C} = \frac{1}{3-1}\int_1^3 2prdr \approx 4p.$

12. A Let $u = e^x - 4$ and $du = e^x dx \Rightarrow \int_{\ln 3}^{\ln 4} e^x(e^x - 4)dx = \int_{-1}^{0} u^2 du = \frac{1}{3}.$

13. B Separating variables, then integrating with $u = 3t^2 - 1$ and $du = 6t\,dt$,

we get $\int \frac{dx}{x} = \int u^{\frac{1}{2}} du \Rightarrow \ln|x| = \frac{2}{3}u^{\frac{3}{2}} + C_1 = \frac{2}{3}\sqrt{(3t^2-1)^3} + C_1 \Rightarrow x(t) = e^{\frac{2}{3}\sqrt{(3t^2-1)^3}+C_1} = Ce^{\frac{2}{3}\sqrt{(3t^2-1)^3}}$.

14. B The quotient of an even and an odd funct.is an odd funct. The symmetric definite integral is ∴ 0.

15. D $v(t) = \int a(t)dt = \int \sin 2t\,dt = \frac{-1}{2}\cos 2t + C \Rightarrow v(0) = \frac{-1}{2} + C \Rightarrow v(t) = \frac{-1}{2}\cos 2t + \frac{1}{2}$.

16. C $f'(x) = 2xe^{x^4}$ and $f''(x) = 2x(4x^3)e^{x^4} + 2xe^{x^4} \Rightarrow f''(0) = 2.$

17. B $A_{Trap} = \frac{7-0}{2\cdot 4}\left[f(0) + 2f\left(\frac{7}{4}\right) + 2f\left(\frac{7}{2}\right) + 2f\left(\frac{21}{4}\right) + f(7)\right] = -53.594$

18. D The average value is $\bar{g} = \frac{1}{\frac{p}{2} - 0}\int_0^{\frac{p}{2}} g(x)dx = \frac{4}{3p}(8 - \sqrt{27}).$

19. D Evaluating the integrals gives $-\ln\left|\cos\frac{5p}{4}\right| - -\ln|\cos p| + \sin p - \sin(-p) = \ln 2 - \ln\sqrt{2}.$

20. B Evaluation of the integral yields the correct answer.

21. A The area is given by $\int_0^2 xe^{x^2}dx$. Letting $u = x^2$, $du = 2x\,dx$ and integrating will give the answer.

22. A Working them out, you can see that B and C converge and A diverges.

23. D The value of g(6) is the area under the curve from 0 to 6. Using basic geometry, you get 18.

24. E Need more info to find the integral of the quotient of 2 functs. than just the value of the 2 integrals.

25. E The integral evaluates to $\frac{x^4}{4} + \frac{e^{3x}}{3} + c$.

26. D Using the formula for volume of disks will give the correct integral.

27. B $\int_{30}^{50} f(x)dx + \int_{50}^{100} f(x)dx = \int_{30}^{100} f(x)dx \Rightarrow \int_{30}^{50} f(x)dx + B = A$, so $\int_{30}^{50} f(x)dx = A - B$.

28. B Using the inverse sine formula and evaluating yields the answer.

29. A Let u = $\frac{1}{t}$, du = $\frac{-1}{t^2}dt$. Substituting back in and integrating we get cos $\frac{1}{t}$. Evaluating, gives the correct answer.

30. C By the Fund. Thm. Of Calc., the integral is arcsinx. Evaluating arcsin(0.4)is 0.412.