

GEMINI
FAMAT State Convention 2001

For all questions, answer E. "NOTA" means none of the above answers is correct.

1. Consider the locus of all points in space which are at a distance of 2 cm. from a given line

segment. The geometric figure thus formed is a

- A. cylinder with a circle at each end B. cylinder with a cone at each end
C. cylinder with a hemisphere at each end. D. cylinder open at each end E.

NOTA

2. Given a right triangle ABC with right angle at C. If $AB = 50$ and the altitude to the hypotenuse is of length 20, what is the length of the shorter leg?

- A. $5\sqrt{5}$ B. $10\sqrt{5}$ C. $15\sqrt{5}$ D. $20\sqrt{5}$ E.

NOTA

3. If $(a - c + 2)x^2 + (a - 2b + 1)x + (a + b - c) = 0$ for all x , then $a + b + c = ?$

- A. -5 B. 0 C. 5 D. 10 E.

NOTA

4. Suppose that triangle ABC has side measures: $AB = 5$, $BC = 12$, and $AC = 13$.

Determine the area, in square units, of the region bounded by the circumcircle and the incircle

of triangle ABC.

- A. $\frac{99p}{4}$ B. $\frac{105p}{4}$ C. $\frac{145p}{4}$ D. $\frac{153p}{4}$ E.

NOTA

5. $\frac{40 + 17\sqrt{5}}{2 + \sqrt{5}} = a + b\sqrt{5}$ where a and b are positive integers. What is the value of $a + b$?

- A. 5 B. 6 C. 8 D. 10 E.

NOTA

6. The sum of a given infinite geometric series is 2001. If the common ratio is $\frac{1}{3}$, what is the first term of the series?

- A. 1002 B. 1221 C. 1228 D. 1334 E.
 NOTA

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7. How many integer values of x satisfy the equation: $\begin{vmatrix} x & 2 & 1 \\ 2 & x & 1 \\ 2 & 1 & x \end{vmatrix} = 0$?

- A. 0 B. 1 C. 2 D. 3 E.
 NOTA

8. To rationalize the denominator the fraction $\frac{1}{\sqrt[3]{a} + \sqrt[3]{b}}$, multiply both numerator and denominator by

- A. $\sqrt[3]{a} - \sqrt[3]{b}$ B. $\sqrt[3]{a^2} - \sqrt[3]{b^2}$ C. $\sqrt[3]{a^2} - \sqrt[3]{ab} + \sqrt[3]{b^2}$ D.
 $\sqrt[3]{a^2} - \sqrt[3]{a^2b^2} + \sqrt[3]{b^2}$
 E. NOTA

9. $\sum_{n=1}^{35} \frac{n}{7} + 2k$

- A. 90 B. 92 C. 160 D. 184 E.
 NOTA

10. When the infinite repeating decimal $0.590\overline{}$ is written as a reduced proper fraction, what is the sum of the numerator and the denominator?

- A. 35 B. 315 C. 1575 D. 1584 E.
NOTA

11. Consider the Pythagorean triplet $\{a, b, c\}$ with $a < b < c$. If $a = 2001$, then the value of b is

- A. 2554 B. 2557 C. 2662 D. 2668 E.
NOTA

12. When $e^{1000} j e^{2001} j$ is expanded, what is the sum of the digits?

- A. 2 B. 5 C. 6 D. 10 E.
NOTA

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13. How many four digit counting numbers are there such that the numbers are less than 5000 and all digits are even?

- A. 125 B. 128 C. 250 D. 268 E.
NOTA

14. A certain inhabited planet has a 40 day year. Suppose five residents of this planet are riding around

in a new four wheel vehicle. What is the probability that at least two of these residents were born on the same day of the year? Round the answer to the nearest hundredth.

- A. 0.23 B. 0.32 C. 0.68 D. 0.77 E.
NOTA

15. To the nearest whole number, how many years will it take \$500.00 to grow to \$1,000.00 if that money is invested at an annual rate of 6%, compounded annually?

- A. 10 B. 12 C. 14 D. 16 E.
NOTA

16. If the sum of two numbers is 1002, and the product of these two numbers is 2001, what is the sum of the reciprocals of these two numbers?

- A. $\frac{1}{2}$ B. $\frac{34}{67}$ C. $\frac{334}{667}$ D. $\frac{3334}{6667}$ E.
NOTA

17. Given the sequence: 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, ... (where the integer n is repeated n times.)

The sum of the first 55 terms of this sequence is a three digit number such that the sum of the ones digit and the hundreds digit equals the tens digit. What is the tens digit?

- A. 4 B. 7 C. 8 D. 9 E.
NOTA

18. What is the area, in square units, of the triangle formed by the points with coordinates (2,4), (8,3), and (-3,12)?

- A. 19.5 B. 21.5 C. 23.5 D. 25.5 E.
NOTA

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19. An aquarium has a square base, one side measuring 12 inches and it is originally filled with

water to a depth of 8 inches. Suppose when a heavy solid cube, with edge 5 inches, is placed in

the water so that it rests on the bottom of the aquarium, the water level rises by one-half inch.

The cube is then removed and some of the water is also removed. When the cube is placed

back in the aquarium, the water level rises by one-fourth inch. What is the depth of the water

at this point?

- A. 2.5 inches B. 4 inches C. 6 inches D. 6.5 inches E.
NOTA

20. How many five letter strings of letters containing exactly two A's can be formed by using the letters from

the word PARABOLA?

- A. 600 B. 800 C. 2400 D. 3600 E.
NOTA

21. A right circular cone and a hemisphere each have a radius r . If the two geometric figures have the

same volume, what is the height of the cone, in terms of r ?

- A. $2r$ B. $3.5r$ C. $4r$ D. $4.5r$ E.
NOTA

22. Three real numbers divide the real number line between $\frac{x}{3}$ and $\frac{2x}{5}$ into four congruent segments.

One of these numbers has the coordinate

A. $\frac{9x}{20}$

B. $\frac{7x}{17}$

C. $\frac{23x}{60}$

D. $\frac{33x}{80}$

E.

NOTA

23. A tank originally contains 5,000 L of pure water. The capacity of the tank is 10,000 L.

Brine that contains 30 grams of salt per L is pumped into the tank at the rate of 25 L per minute.

What is the number of grams of salt per L at the end of one hour. Round answer to the nearest tenth.

A. 5.8

B. 6.9

C. 7.1

D. 8.3

E.

NOTA

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24. What is the number of positive integral divisors of 2001?

A. 4

B. 6

C. 8

D. 10

E.

NOTA

25. When 2001^{21} is expanded, the last four digits are

A. 0001

B. 2001

C. 4001

D. 6001

E.

NOTA

26. The graph of the parametric equations $x = 2^t$ and $y = 2^{-t}$ is a part of a
A. line B. circle C. parabola D. hyperbola E.
NOTA

27. The centroid of the triangle with vertices at (3,4), (9,1) and (-6, 7) has coordinates
A. (2,4) B. (2,5) C. (3,3) D. (3,5) E.
NOTA

28. A particle moves along a line with velocity $v(t) = 3t^2 - 12t + 9$ where t represents time and $t > 0$.
For what time t is the particle moving to the left?
A. for no t B. $1 < t < 3$ C. $t > 1$ D. $t > 3$ E.
NOTA

29. A function $g(n)$ is defined over the non-negative integers n as follows: $g(0) = -1$;
 $g(1) = 1$;
and $g(n+2) = g(n) - 2g(n+1)$. The value of $g(6)$ is
A. -99 B. -41 C. 41 D. 99 E.
NOTA

30. Seven digit numbers are formed by permuting the digits 1, 2, 3, 4, 5, 6, 7.
If all permutations are equally likely, what is the probability that the odd digits will
occur
in increasing order?
A. $\frac{1}{24}$ B. $\frac{1}{12}$ C. $\frac{1}{6}$ D. $\frac{1}{4}$ E.
NOTA

