

<p><b>1</b></p> $\text{cis } 60^\circ = \frac{1}{2} + \frac{\sqrt{3}}{2}i$ $2 \text{ cis } 60^\circ = 1 + \sqrt{3}i$ <p style="text-align: right;">(C)</p>	<p><b>2</b></p> $(1+i)(1+i) = 2i$ $(2i)^2 = -4$ <p style="text-align: right;">(A)</p>	<p><b>3</b></p> $ 6-8i  =$ $\sqrt{6^2+8^2} = 10$ <p style="text-align: right;">(B)</p>								
<p><b>4</b></p> $z = 1+i \quad \bar{z} = 1-i$ $(1+i)(1-i) + 3(1+i) - (1-i)^2 - 2(1-i)$ $= 1+1+3+3i - (1-2i+i^2) - 2+2i$ $= 5+3i+2i-2+2i$ $= 3+7i$ <p style="text-align: right;">(C)</p>	<p><b>5</b></p> $\begin{vmatrix} 2i & 3i \\ -1 & -i \end{vmatrix} - \begin{vmatrix} 0 & 3 \\ -1 & -i \end{vmatrix} + \begin{vmatrix} 0 & 3 \\ 2i & 3i \end{vmatrix}$ $= 2+3i-3+(-6i)$ $= -1-3i$ <p style="text-align: right;">(D)</p>	<p><b>6</b></p> $\frac{1}{1+i} \cdot \frac{1-i}{1-i} = \frac{1-i}{2} \quad \frac{1}{\frac{1-i}{2}} = \frac{2}{1-i}$ $\frac{2}{1-i} \cdot \frac{1-i}{1-i} = \frac{2-2i}{2} = 1-i$ $1-i+i=1 \quad x=1$ <p style="text-align: right;">(B)</p>								
<p><b>7</b></p> $(1+i)(1+i) = 2i$ $(1+i)^{17} = (2i)^8(1+i) =$ $256(1+i) =$ $256+256i$ <p style="text-align: right;">(E)</p>	<p><b>8</b></p> $\sqrt{2}i = (\sqrt{2})(i)^{\frac{1}{2}}$ $(0+i)^{\frac{1}{2}} = (\text{cis } 90^\circ)^{\frac{1}{2}} = \text{cis } 45^\circ$ $(\sqrt{2})(\cos 45^\circ + i \sin 45^\circ) =$ $(\sqrt{2})\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = 1+i$ <p style="text-align: right;">(B)</p>	<p><b>9</b></p> $(1+3i)^4$ $(1+3i)(1+3i) = 1+6i-9 = -8+6i$ $(-8+6i)^2 = 64-96i-36$ $= 28-96i$ <p style="text-align: right;">(B)</p>								
<p><b>10</b></p> $(2+3i)(7-4i) =$ $14+13i+12 = 26+13i$ <p>Quadrant I</p> <p style="text-align: right;">(A)</p>	<p><b>11</b></p> $i^{2001} = ?$ $\frac{2001}{4} = 500 \text{ R } 1$ $i^{2001} = i$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>1</td><td>i</td></tr> <tr><td>2</td><td>-1</td></tr> <tr><td>3</td><td>-i</td></tr> <tr><td>4</td><td>1</td></tr> </table> <p style="text-align: right;">(C)</p>	1	i	2	-1	3	-i	4	1	<p><b>12</b></p> $(2 \text{ cis } 80^\circ)\left(\frac{1}{3} \text{ cis } 40^\circ\right)$ $= 4 \text{ cis } 120^\circ$ $= 4(\cos 120^\circ + i \sin 120^\circ)$ $= 4\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -2+2\sqrt{3}i$ <p style="text-align: right;">(B)</p>
1	i									
2	-1									
3	-i									
4	1									
<p><b>13</b></p> $f(-i) = f(-1 \cdot i), x = -1$ $2i(1)^3 - 4(-1)^2 + 5i(-1) - 3$ $= -2i - 4 - 5i - 3$ $= -7-7i$ <p style="text-align: right;">(A)</p>	<p><b>14</b></p> $(3+i)^3 + a(3+i)^2 + b(3+i) + c = 0$ $18+26i+a(8+6i)+b(3+i)+c=0$ $9a+6ia+3b+6i+c = -18-26i$ $9a+3b = -18$ $6a+3b = -26$ $a = -\frac{20}{3} \quad b = 14 \quad \frac{a}{b} = \frac{-10}{21}$ <p style="text-align: right;">(B)</p>	<p><b>15</b></p> $x^2 = 8 \quad  1+i\sqrt{3}  = 2$ $x^2 - 8 = 0 \quad  1+i\sqrt{3}  = 2$ $(x-2)(x^2+2x+4) = 0 \quad  2i  = 2$ $x = 2$ $\frac{-2 \pm \sqrt{4-4(4)}}{2} = 1 \pm i\sqrt{3}$ $ 2+ 2+i\sqrt{3}  +  1-i\sqrt{3}  = 6$ <p style="text-align: right;">(B)</p>								

<p>16</p> $\frac{1+i\sqrt{5}}{\sqrt{3+i}} \cdot \frac{\sqrt{3-i}}{\sqrt{3-i}} =$ $\frac{\sqrt{5} + \sqrt{3}}{2} \approx 1.984$ <p style="text-align: right;">A</p>	<p>17</p> $\prod_{k=1}^{200} \frac{ki}{k+2} = \left(\frac{i}{3}\right) \left(\frac{2i}{4}\right) \left(\frac{3i}{5}\right) \dots \left(\frac{200i}{202}\right)$ $= \frac{200! (i^{200})}{\frac{202!}{2}} = \frac{2 \cdot i^{200}}{201 \cdot 202} = \frac{1}{20301}$ <p style="text-align: right;">C</p>	<p>18</p> $2e^{\frac{\pi}{3}i} = 2 \operatorname{cis} \frac{\pi}{3} =$ $2(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}) =$ $1 + \sqrt{3}i$ <p style="text-align: right;">E</p>
<p>19</p> <p style="text-align: center;">I + III</p> <p>I. <math>(1+i)(1-i) = 2 \checkmark</math>              II. <math>2+i+(2-i) = 4</math> (not imag.)              III. <math>\frac{1}{i} \cdot \frac{-i}{-i} = -i \checkmark</math></p> <p style="text-align: right;">D</p>	<p>20</p> $\frac{1}{2-3i} \cdot \frac{2+3i}{2+3i} = \frac{2+3i}{13}$ <p style="text-align: right;">A</p>	<p>21</p> <p>Descartes's Rule of Signs</p> <p>0 real + 6 complex</p> <p style="text-align: right;">A</p>
<p>22</p> $b^2 - 4ac < 0$ $9 - 4(4)(k-1) < 0$ $9 - 16k + 16 < 0$ $25 < 16k$ $k > \frac{25}{16}$ <p style="text-align: right;">D</p>	<p>23 <math>a=i \quad n=25</math></p> $r = \frac{-2}{-1+i} = 1+i$ $S = \frac{a(1+r)^n}{1-r} = \frac{i(1+(1+i))^{25}}{1-(1+i)}$ $\frac{i(-4095 - 4096i)}{-i} = 4095 + 4096i$ <p style="text-align: right;">A</p>	<p>24</p> <p>prod. of all roots = <math>\sum a</math></p> $-3 = (i)(-i)(a)(b) = -3$ $(a)(b) = -3$ <p style="text-align: right;">A</p>
<p>25</p> $x^{\frac{4}{3}} + 15x^{\frac{1}{3}} - 16 = 0$ $(x^{\frac{2}{3}} - 1)(x^{\frac{2}{3}} + 16) = 0$ $x^{\frac{2}{3}} = 1 \quad x^{\frac{2}{3}} = -16$ $x = \pm 1 \quad x = \pm 64i$ <p style="text-align: right;">B</p>	<p>26</p> $\frac{6 \operatorname{cis} \frac{3\pi}{4}}{3 \operatorname{cis} \frac{\pi}{6}} = \frac{2(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4})}{\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}}$ $= \frac{-\sqrt{2} + \sqrt{2}i}{2} \cdot 2 = \frac{-\sqrt{2} + 2i}{\sqrt{3} + i} \cdot \frac{\sqrt{3} - i}{\sqrt{3} - i}$ $= \frac{-\sqrt{2} - \sqrt{6} + (\sqrt{2} + \sqrt{6})i}{2}$ <p style="text-align: right;">E</p>	<p>27</p> $\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^{4i} = (\operatorname{cis} \frac{\pi}{4})^{4i}$ $\operatorname{cis} \frac{\pi}{4} = e^{i\frac{\pi}{4}}$ $(e^{i\frac{\pi}{4}})^{4i} = e^{-\pi}$
<p>28</p> $f(x) = \frac{-3x}{1+x} \quad f(a) = \frac{9+3i}{4}$ $\frac{3a}{1+a} = \frac{9+3i}{4} \quad 12a = (9+3i)(1+a)$ $12a = 9+3i+9a+3ia$ $3a-3ia = 9+3i$ $a(3-3i) = 9+3i$ $a = \frac{3+i}{1-i} \cdot \frac{1+i}{1+i} = \frac{1+4i}{2}$ <p style="text-align: right;">D</p>	<p>29</p> $(\sqrt{3} + i)^4$ $((\sqrt{3} + i)(\sqrt{3} + i))^2 = (2 + 2\sqrt{3}i)^2$ $= 4(1 + \sqrt{3}i)(1 + \sqrt{3}i) = -8 + 8\sqrt{3}i$ $= -16\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) = -16 \operatorname{cis} 300^\circ$ <p style="text-align: right;">C</p>	<p>30</p> $\cosh(\pi i) = \frac{e^{\pi i} + e^{-\pi i}}{2} = \frac{-1 + (-1)}{2} = -1$ $\sinh(\pi i) = \frac{e^{\pi i} - e^{-\pi i}}{2} = 0$ $\cosh^2(\pi i) + \sinh^2(\pi i) = 2$ <p style="text-align: right;">C</p>