

TEAM QUESTION #1  
Alpha Bowl 2001

What is the largest rational root of the equation  $6x^{\frac{1}{2}} - 31x^{\frac{3}{2}} + 35x^{\frac{5}{2}} = 0$

TEAM QUESTION #2  
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Determine the exact value of  $\sin 195^\circ$

Write the answer in the form  $\frac{\sqrt{a} - \sqrt{b}}{c}$  where a, b, and c are integers.

TEAM QUESTION #3  
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Consider the right triangle with vertices: (0,0), (0,7), and (10,0).

M = the volume of the solid formed by rotating the triangle about the Y-axis.

N = the volume of the solid formed by rotating the triangle about the X-axis.

What is the exact value of M - N?

TEAM QUESTION #4  
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What is the largest value of x, given in degrees, for  $0^\circ \leq x \leq 90^\circ$ , that satisfies the equation:  $\cos 2x \cos x - \sin 2x \sin x + \sin x - \cos x = 0$  ?

TEAM QUESTION #5

Alpha Bowl 2001

Given:  $P(x) = ax^5 + bx^4 + cx^3 + dx^2 + f$ ,  $a, b, c, d$ , and  $f$  are integers.

- True/false
1. \_\_\_ If  $a \neq 0$ ,  $P(x)$  is a polynomial of degree 5.
  2. \_\_\_ If  $a \neq 0$ , the sum of the roots of  $P(x)$  is  $-b$ .
  3. \_\_\_ If  $a \neq 0$ , the product of the roots of  $P(x)$  is  $f/a$ .
  4. \_\_\_  $P(x)$  has at least one real root.
  5. \_\_\_  $P(x)$  has exactly one Y-intercept.
  6. \_\_\_ If  $b=d=0$ , and  $a, c$ , and  $f$  are not 0, then  $P(x)$  is an odd function.
  7. \_\_\_ If  $a=c=0$ , and  $b, d$ , and  $f$  are not 0, then  $P(x)$  is an even function.
  8. \_\_\_ The graph of  $P(x)$  has at least one asymptote.
  9. \_\_\_ The graph of  $P(x)$  is a smooth continuous curve.

A = the number of true statements.

B = the number of false statements.

What is the value of  $(A)(B)$ ?

TEAM QUESTION #6

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Given an arithmetic sequence with first three terms:

$$\frac{1}{1 + \sqrt{x}}, \quad \frac{1}{1 - x}, \quad \frac{1}{1 - \sqrt{x}}$$

What is the 2001st term?

TEAM QUESTION #7

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The line with equation  $3x + 5y = 47$  is tangent to a circle with center at  $(1,2)$ .

What are the coordinates of the point of tangency?

TEAM QUESTION #8  
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A is the solution set of the inequality:  $x^3 - 13x^2 + 47x - 35 \leq 0$

B is the solution set of the inequality:  $\frac{x^2 - 5x + 6}{x - 6} \geq 0$

What is  $A \cap B$ ?

TEAM QUESTION #9  
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Given:  $f(x) = \begin{cases} x^2 & \text{for } x < 1 \\ 3x & \text{for } x \geq 1 \end{cases}$

$$g(x) = \begin{cases} x & \text{for } 0 \leq x \leq 5 \\ \sqrt{x - 9} & \text{for } x > 5 \end{cases}$$

$$h(x) = \begin{cases} -x + x^2 & \text{for } x < 0 \\ -x + 3x^2 & \text{for } x \geq 0 \end{cases}$$

$$A = f(g(h(0))) \quad \text{and} \quad B = f(h(g(2))) \quad \text{and} \quad C = h(g(f(-5)))$$

What is the value of  $A + B - C$  ?

TEAM QUESTION #10  
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A Box of marbles contains 4 red marbles and the rest are black. If there exists 1001 permutations of the marbles, how many are black?

TEAM QUESTION #11  
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Determine the coordinates of the two points on the Y-axis that are 10 units from the graph of the line with equation  $12x - 5y = 10$ .

TEAM QUESTION #12  
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Consider the graph of the ellipse with equation:  $\frac{x^2}{169} + \frac{y^2}{25} = 1$ .

Suppose a rectangle is inscribed in this ellipse and one pair of opposite sides of the rectangle contain the foci of the ellipse.

What is the area, in square units, of this rectangle?  
Round the answer to the nearest tenth.

TEAM QUESTION #13  
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Find all values of  $x$  which satisfy the equation:

$$|x + 9| + |x - 2| = 11.$$

TEAM QUESTION #14  
Alpha Bowl 2001

The graphs of the lines with equations  $3x - 4y = 10$  and  $7x + 12y = 3$  intersect and form two acute angles and two obtuse angles. What is the measure of one of the acute angles. Round the answer to the nearest degree.

TEAM QUESTION #15  
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Given: 
$$\begin{vmatrix} b + c & a & a \\ b & c + a & b \\ c & c & a + b \end{vmatrix} = 1540$$

If  $a$ ,  $b$ , and  $c$  are positive integers and  $a < b < c$ , what is the value of  $c$ ?

