1. $\int_1 \sqrt{1+\left(\frac{x^2}{4}-\frac{1}{x^2}\right)^2}\,dx = \int_1 \left(\frac{1}{4}x^2+x^{-2}\right)dx = \frac{x^3}{12}-\frac{1}{x}\Big] = \boxed{\frac{\ }{12}}$ C

2. $u=e^x \quad dv=\cos x\,dx \qquad e^x\sin x-\int e^x\sin x\,dx$
$du=e^x dx \quad v=\sin x \qquad u=e^x \quad dv=\sin x\,dx$
$du=e^x dx \quad v=-\cos x$
$-e^x\cos x+\int e^x\cos x\,dx$

$\int e^x\cos x\,dx = e^x\sin x-\left(-e^x\cos x+\int e^x\cos x\,dx\right)$
$2\int e^x\cos x\,dx = e^x\sin x+e^x\cos x$
$\int_0^{\pi/2} e^x\cos x = \frac{e^x\sin x+e^x\cos x}{2}\Big]_0^{\pi/2} = \frac{e^{\frac{\pi}{2}}}{2}-\frac{1}{2}$ A

3. $\int_0^{\pi/4} \tan^2 x(\tan^2 x+1)\sec^2 x\,dx = \int_0^{\pi/4}(\tan^4 x+\tan^2 x)\sec^2 x\,dx =$
$\int_0^1 (u^4+u^2)\,du = \frac{1}{5}u^5+\frac{1}{3}u^3\Big]_0^1 = 8/15$ D

4. $\int_2^3 \frac{2}{x^2-1}dx = \int_2^3\frac{1}{x-1}dx+\int_2^3\frac{-1}{x+1}dx = \ln\left|\frac{x-1}{x+1}\right|\Big]_2^3 = \ln\frac{3}{2}$ E

5. $\ln y = \frac{\ln(1+3x)}{2x} \qquad \lim_{x\to 0}\frac{\ln(1+3x)}{2x} = \lim_{x\to 0}\frac{3}{1+3x}\cdot\frac{1}{2} = \frac{3}{2}$
$y=e^{3/2}$ B

6. $\int_1^\infty \frac{x}{(1+x^2)^2}dx = \frac{1}{2}\int_2^\infty u^{-2}du = \lim_{b\to\infty} -\frac{1}{2u}\Big]_2^b = \lim_{b\to\infty}\left(-\frac{1}{2b}-\frac{-1}{4}\right) = 1/4$ B

7. $e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots \qquad e^x-1 = x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$
$\frac{e^x-1}{x} = 1+\frac{x}{2!}+\frac{x^2}{3!}+\cdots$ D

8. I. Converges by A.S. Test II. $\left|\frac{\sin\sqrt{n}}{\sqrt{n^3+1}}\right| \le \frac{1}{\sqrt{n^3+1}}$ converges
D absolutely by Comparison III. Converges by Integral Test

9. $\lim_{n\to\infty}\left|\frac{x^{n+1}}{\sqrt{n+1}}\cdot\frac{\sqrt{n}}{x^n}\right| = \lim_{n\to\infty}|x|$; $|x|<1$; Let $x=1$ $\sum_{n=1}^\infty \frac{1}{\sqrt{n}}$ diverges
Let $x=-1$ $\sum_{n=1}^\infty \frac{(-1)^n}{\sqrt{n}}$ converges by A.S. Test B

10. $\frac{1}{1+x} = 1-x+x^2\cdots \quad \frac{d}{dx}\left(\frac{1}{1+x}\right) = -\frac{1}{(1+x)^2} = -1+2x-3x^2+\cdots$
$\frac{1}{(1+x)^2} = 1-2x+3x^2\cdots$ A

11. $\sum_{n=1}^\infty\left(\frac{1}{2^{n+1}}-\frac{1}{3^{n+1}}\right) = \frac{\frac{1}{4}}{1-\frac{1}{2}}-\frac{\frac{1}{9}}{1-1/3} = \frac{1}{2}-\frac{1}{6} = \frac{1}{3}$ B

12. $\sin(x^2)\approx x^2-\frac{x^6}{3!}+\frac{x^{10}}{5!}$
$\int_0^1 \sin(x^2)\,dx \approx \frac{x^3}{3}-\frac{x^7}{7\cdot 3!}+\frac{x^{11}}{11\cdot 5!}\Big]_0^1 = \frac{1}{3}-\frac{1}{42}+\frac{1}{1320}\approx .31028$ C

13. at $(-2,-5)$, $t=1$ $\quad \frac{dy}{dx} = \frac{dy}{dt}\Big/\frac{dx}{dt} = \frac{2t-5}{3t^2-3}$
$\frac{dy}{dx}\Big|_{t=1} = \frac{-3}{0}$ undefined D

14. $\frac{1}{2}\int_{-\pi/4}^{\pi/4} 4\cos 2\theta\,d\theta = 4\int_0^{\pi/4}\cos 2\theta\,d\theta = 2\sin 2\theta\Big]_0^{\pi/4} =$
2 B

15. $L = \int_0^{2\pi}\sqrt{(1-\cos t)^2+(\sin t)^2}\,dt = \int_0^{2\pi}\sqrt{2-2\cos t}\,dt =$
$\sqrt{2}\int_0^{2\pi}\sqrt{1-\cos t}\,dt = \sqrt{2}\int_0^{2\pi}\sqrt{2\sin^2\frac{t}{2}}\,dt = 2\int_0^{2\pi}\left|\sin\frac{t}{2}\right|dt$
$= -4\cos\left(\frac{t}{2}\right)\Big]_0^{2\pi} = 8$ D [Note: $\sin\frac{t}{2}\ge 0$ since $0\le\frac{t}{2}\le\pi$]

$\ldots 4\sin 3\theta\sin\theta$; $\frac{dy}{d\theta} = 4\sin 3\theta\cos\theta + 12\cos 3\ldots$

$A+\theta = \frac{\pi}{4}$ $\frac{dx}{d\theta} = -4\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}+12\cdot-\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} = -2\ldots$
$\frac{dy}{d\theta} = 4\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}+12\cdot-\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2} = 2-6\ldots$
$\frac{dy}{d\theta}\Big/\frac{dx}{d\theta} = \frac{-4}{-8} = 1/2$ C

17. $f(x)=\tan x$; $\tan\pi/4 = 1$; $f'(x)=\sec^2 x$, $f'(\pi/4\ldots$
$f''(x) = 2\sec^2 x\tan x$; $f''(\pi/4) = 4$
$\tan x = 1+2(x-\pi/4)+\frac{4(x-\frac{\pi}{4})^2}{2}$ A

18. at $(1,1)$, $t=0$. $r'(t) = \langle 2e^{2t}, -e^{-t}\rangle$
$r'(0) = \langle 2,-1\rangle$ A

19. $=\int_0^1 e^{2x}dx = \frac{1}{2}e^{2x}\Big]_0^1 = \frac{1}{2}e^2-\frac{1}{2}$ D

20. $r'(t) = \langle 2,8t\rangle \quad r'(1) = \langle 2,8\rangle \quad \|\langle 2,8\ldots$

21. $2x(y+3)+(x^2-4)\frac{dy}{dx} = 0$; $-\frac{dy}{y+3} = \frac{2x}{x^2-4}dx\ldots$
$-\ln|y+3| = \ln|x^2-4|+C_1$; $\ln|y+3| = -\ln|x\ldots$
$\ln|y+3| = \ln\frac{1}{|x^2-4|}+C_2$; $|y+3| = C_3\left|\frac{1}{x^2\ldots}\right.$
$y+3 = \frac{C_3}{x^2-4}$; $y = \frac{C_3}{x^2-4}-3$ A

22. $\frac{dy}{dx}+\frac{5}{x}\cdot y = -3x^3$; $e^{\int\frac{5}{x}dx} = e^{5\ln|x|} = \ldots$
$x^5\frac{dy}{dx}+5x^4y = -3x^8$ (Note: Mult by $x^5\ldots$
gives same result) $\frac{d}{dx}(x^5y) = -3x^8\ldots$
$x^5y = -\frac{x^9}{3}+C \qquad y = -\frac{x^4}{3}+\frac{C}{x^5}$ D

23. $g(x) = \ln|x|$ or $\ln x$ since $x>0$; $g(x^3) = \int_1^{x^3}\frac{1}{t}\ldots$

24. $\frac{1-\cos t}{t^2} = \frac{1}{2!}-\frac{t^2}{4!}+\frac{t^4}{6!}-\frac{t^6}{8!}+\cdots$
$\int_0^x \frac{1-\cos t}{t^2}dt = \frac{1}{2!}x-\frac{x^3}{3\cdot 4!}+\frac{x^5}{5\cdot 6!}-\frac{x^7}{7\cdot 8!}+\frac{x^9}{9\cdot 1\ldots}$

25. Diverges E $\int_0^3\frac{1}{(x-3)^2}dx$ diverges

26. $\frac{1}{2}\int_3^\infty\left(\frac{1}{x-1}-\frac{1}{x+1}\right)dx = \lim_{b\to\infty}\frac{1}{2}\ln\left(\frac{x-1}{x+1}\right)\Big]_3^b = C\ldots$

27. $\ln x = (x-1)-\frac{(x-1)^2}{2}+\frac{(x-1)^3}{3}-\frac{(x-1)^4}{4c^4}$ for $1\ldots$
Max value of $\left|\frac{(x-1)^4}{4c^4}\right|$ is $< \frac{.3^4}{4} = .002025\ldots$

28. $\frac{dy}{dt} = 6t$; $\frac{dx}{dt} = 3t^2$; $\frac{dy}{dx} = \frac{6t}{3t^2} = \frac{2}{t} = \frac{2}{3\sqrt{x}\ldots}$

29. $u = x^3 \quad du = 3x^2dx \qquad \frac{1}{3}\int_0^1\frac{1}{u^2+4}du = \frac{1}{6}\arctan\frac{x^3}{2}\Big]_0^1 = \ldots$

30. $V = 2\pi\int_2^3\frac{x}{(x-1)(4-x)}dx = 2\pi\left(\frac{1}{3}\int_2^3\frac{1}{x-1}dx+\ldots\right.$
$= 2\pi\left(\frac{1}{3}\ln(x-1)\Big|_2^3+\frac{4}{3}\ln(4-x)\Big|_2^3\right) = \cdot$
$2\pi\left(\frac{1}{3}\ln 2+4/3\ln 2\right) = 2\pi\cdot\frac{5}{3}\ldots$