

ALPHA Individual Test
FAMAT State Convention 2001

For all questions, answer E. "NOTA" means none of the above answers is correct.

1. Given triangle ABC with $AB = 7$, $BC = 8$ and $AC = 13$. What is the degree measure of the largest angle of the triangle?

- A. 90 B. 120 C. 145 D. 150 E. NOTA

2. If $f(x) = \frac{1}{x+2} + 1$, then $f^{-1}(x) =$

- A. $\frac{3-2x}{x-1}$ B. $\frac{3-x}{x-2}$ C. $x+3$ D. $x+1$ E. NOTA

3. Determine the value of a such that $(a, a+2)$, $(-2, 4)$, and $(7,10)$ are collinear.

- A. -8 B. -4 C. 8 D. 10 E. NOTA

4. One of the asymptotes of the hyperbola with equation $\frac{(x-a)^2}{a^2} - \frac{(y-b)^2}{b^2} = 1$ has a Y-intercept of

- A. $-2a$ B. $-2b$ C. $2a$ D. $2b$ E. NOTA

5. If $x = 2 \tan \mathbf{q}$, then $\sqrt{(4+x^2)^3}$ can be expressed as

- A. $2|\sec \mathbf{q}|$ B. $4|\sec^3 \mathbf{q}|$ C. $4\sec^2 \mathbf{q}$ D. $8|\sec^3 \mathbf{q}|$ E. NOTA

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6. For any given parabola, the ratio of the focal distance to the length of the latus rectum is

- A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. $\frac{1}{1}$ D. no constant ratio exists E. NOTA

7. What is the name of the polar graph with equation $r = 3 + 3 \cos \theta$?
- A. rose B. lemniscate C. cissoid D. spiral E. NOTA
8. Where defined, $\frac{x^{-2} - y^{-2}}{x^{-4} - y^{-4}} =$
- A. $\frac{x^2 y^2}{y^2 - x^2}$ B. $x^2 + y^2$ C. $\frac{x^2 y^2}{x^2 + y^2}$ D. $\frac{x^2 - y^2}{x^2 y^2}$ E. NOTA
9. A particle moves along a path defined parametrically by $x = \sec pt$ and $y = \tan pt$ for $0 \leq t \leq \frac{1}{2}$. The path of motion is along a part of a
- A. line B. circle C. parabola D. hyperbola E. NOTA
10. The amplitude of the graph of $f(x) = 12 \sin x - 5 \cos x$ is
- A. 7 B. 13 C. 17 D. 60 E. NOTA
11. Given the points with coordinates A: (0,8), B: (3,-1), and C: (9,4). The point P, on the X-axis which minimized $(PA)^2 + (PB)^2 + (PC)^2$ is
- A. (2,0) B. (4,0) C. (5,0) D. (6,0) E. NOTA
12. $\sum_{n=1}^{\infty} \frac{n}{3^n} =$
- A. $\frac{1}{2}$ B. $\frac{2}{3}$ C. $\frac{3}{4}$ D. $\frac{4}{5}$ E. NOTA

13. Suppose $\frac{17n-23}{4n^2+2n-6}$ is decomposed into partial fractions $\frac{A}{4n-1} + \frac{B}{n+6}$. $A+B =$
- A. 1 B. 2 C. 3 D. 8 E. NOTA
14. The base of a pendulum 30 inches long hangs 5 inches above the horizontal when the pendulum is perfectly still. If it swings 30 degrees from the vertical, how high above the horizontal is the base of the pendulum at the highest point of this 30 degree swing?
Round the answer to the nearest tenth.
- A. 5.4 in. B. 6.5 in. C. 7.8 in. D. 9.0 in. E. NOTA
15. Find the maximum value of xy if $x + \frac{4}{y} = 1$ and $y + \frac{4}{x} = 25$.
- A. 1 B. 4 C. 16 D. 25 E. NOTA
16. If $f(x) = x^3$ then $\frac{f(x+2) - f(x)}{2} =$
- A. 4 B. $3x^2 + 3x + 4$ C. $3x^2 + 6x + 4$ D. $3x^2 + 6x + 8$ E. NOTA
17. Suppose a cylinder is inscribed in a sphere of radius 6 cm. If the radius of the cylinder is $2\sqrt{3}$ cm., find the number of cubic centimeters in the volume of the sphere which is outside the cylinder.
Round the answer to the nearest tenth.
- A. 302.4 B. 424.9 C. 535.4 D. 720.1 E. NOTA
18. Consider the polynomial $P(x) = x^4 + ax^3 + bx^2 + c$. If $P(2) = 1$, $P(3) = 11$, and P is an even function, then what is the value of $a + b + c$?
- A. 11 B. 18 C. 22 D. 25 E. NOTA

19. Given: $h(x) = \log(6 - |2x^2 - 4|)$, $g(x) = 10^x$, and $f(x) = x - 6$.

$f(g(h(a)))$ is defined so long as

- A. $|a| < 1$ B. $|a| < \sqrt{2}$ C. $|a| < 2$ D. $|a| < \sqrt{5}$ E. NOTA

20. What is the value of the term in the expansion of $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^9$ which does not contain an x is

- A. $\frac{7}{18}$ B. $\frac{4}{9}$ C. 3 D. 12 E. NOTA

21. What is the distance from the point $(-8, 30)$ to the line with slope $\frac{3}{4}$ and y -intercept 5?

- A. 24.8 B. 25 C. 28.5 D. 30.5 E. NOTA

22. A 12 meter tower casts a 15 meter shadow directly down a hill when the angle of elevation of the sun is 47 degrees. Determine the angle of elevation of the hill to the nearest degree.

- A. 10 B. 11 C. 13 D. 14 E. NOTA

23. The number of ordered pairs, (x, y) which satisfy the system

$x^2 - xy + y^2 = 6$ and $2x - xy + 2y = -1$ is infinite. These points lie on a

- A. line B. circle C. parabola D. hyperbola E. NOTA

24. Given $F(1) = 3$ and for all n greater than 1, $F(n) = 2F(n-1) + 1$. What is the largest n such that $F(n)$ is less than 2001?

- A. 9 B. 10 C. 11 D. 12 E. NOTA

25. What is the number of values of x such that $0^\circ \leq x < 360^\circ$ which satisfies the equation $\sin 6x = 0.5$?

- A. 2 B. 4 C. 6 D. 12 E. NOTA

26. Determine the value of the determinant of the product of the given matrices:

$$\begin{pmatrix} 3 & 0 & 4 \\ 2 & 1 & -2 \end{pmatrix} \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$$

- A. -20 B. 20 C. 100 D. 120 E. NOTA

27. If $\tan 2x = \frac{3}{4}$ for $0 < x < \frac{\pi}{2}$, then the value of $\sin x$ is

- A. $\frac{1}{8}$ B. $\frac{1}{4}$ C. $\frac{1}{2\sqrt{3}}$ D. $\frac{1}{\sqrt{10}}$ E. NOTA

28. The number of asymptotes for the graph of $f(x) = \frac{x^2 - 5}{x - 2}$ is exactly

- A. 0 B. 1 C. 2 D. 3 E. NOTA

29. The value of $\sum_{n=2}^{2000} \log \frac{n+1}{n}$ correct to the nearest hundredth is

- A. 3.00 B. 3.21 C. 3.30 D. 3.42 E. NOTA

30. In a certain game, a fair coin is tossed until a head appears, or until it has been tossed three times. Given that a head does not occur on the first toss, what is the probability that the coin will be tossed three times?

- A. $\frac{1}{16}$ B. $\frac{1}{8}$ C. $\frac{1}{4}$ D. $\frac{1}{2}$ E. NOTA

