

FAMAT STATE CONVENTION 2001
ALPHA INDIVIDUAL SOLUTIONS

<p>1. B. $169=49+64-(2)(7)(8)\cos x$</p> <p>$\cos x = -0.5$ $x = 120$ degrees</p>	<p>2. A. $x = \frac{1}{y+2} + 1$; $xy + 2x = 1 + y + 2$</p> <p>$y(x-1) = -2x + 3$ $y = (3 - 2x)/(x-1)$</p>
<p>3. D. $(a+2-4)/(a+2) = 2/3$ $3a - 6 = 2a + 4$ $a = 10$</p>	<p>4. D. $(y-b)/(x-a) = -b/a$ $ay - ab = -bx + ab$ $ay = -bx + 2ab$</p>
<p>5. D. $\sqrt{(4 + 4\tan^2 q)^3} = 8\sqrt{(1 + \tan^2 q)^3}$ $= 8 \sec^3 q$</p>	<p>6. A.</p>
<p>7. E. The equation defines a cardioid.</p>	<p>8. C. Multiply numerator and denominator by $x^4 y^4$</p> $\frac{x^2 y^4 - y^2 x^4}{y^4 - x^4} = \frac{x^2 y^2 \mathbf{i}^2 - x^2 \mathbf{j}^2}{(y^2 + x^2)(y^2 - x^2)}$
<p>9. D. $x^2 = \sec^2 p t$; $y^2 = \tan^2 p t$ $x^2 - y^2 = 1$</p>	<p>10. B. For $f(x) = A \sin x \pm B \cos x$ The amplitude is $\sqrt{A^2 + B^2}$</p>
<p>11. B. Let $P = (x,0)$</p> <p>$(PA)^2 = x^2 + 64$; $(PB)^2 = (x-3)^2 + 1$</p> <p>$(PC)^2 = (x-9)^2 + 16$</p> <p>Sum = $3x^2 - 24x + 171$</p> <p>Sum = $3(x-4)^2 + 123$</p> <p>Minimum for $x = 4$</p>	<p>12. C. $S = \frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \frac{4}{81} + \frac{5}{243} + \dots$</p> <p>$3S = 1 + \frac{2}{3} + \frac{3}{9} + \frac{4}{27} + \frac{5}{81} + \dots$</p> <p>Subtract: $2S = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots$</p> $2S = \frac{1}{1 - \frac{1}{3}} = \frac{3}{2}$ $S = \frac{3}{4}$
<p>13. B. $A(n+6) + B(4n-1) = 17n - 23$ When $n=-6$: $-25B = -125$; $B = 5$ When $n=0.25$: $6.25A = -18.75$; $A = -3$</p>	<p>14. D. Use 30-60-90 right triangle with hypotenuse 30. and top angle 30 degrees. Altitude is $15\sqrt{3}$. Height from horizontal is $35 - 15\sqrt{3} = 9.0$</p>
<p>15. C. $xy + 4 = y$; $xy + 4 = 25x$</p> <p>$y = 25x$; $25x^2 - 25x + 4 = 0$ $(5x - 4)(5x - 1) = 0$; $x = 4/5$ or $x = 1/5$ $(x,y) = (4/5, 20)$ or $(1/5, 5)$ Maximum product $xy = 16$</p>	<p>16. C. $\frac{x^3 + 6x^2 + 12x + 8 - x^3}{2}$</p> <p>$3x^2 + 6x + 4$</p>

17. C. Volume sphere = $\frac{4}{3}(p)(6^3)$
 Volume cylinder = $(p)(12)(4\sqrt{6})$
 Difference: 535.4

18. B. Since P is even a = 0
 $16 + 4b + c = 1$; $81 + 9b + c = 11$
 $4b + c = -15$; $9b + c = -70$
 b = -11 and c = 29. a+b+c=18

19. D. $|2a^2 - 4| < 6$; $-6 < 2a^2 - 4 < 6$
 $-1 < a^2 < 5$; $|a| < \sqrt{5}$

20. A. Seventh term : $\frac{7}{18}$

21. A. Equation: $3x-4y+20=0$
 $d = \frac{|-24 - 120 + 20|}{5} = 24.8$

22. D. $(\sin 43)/15 = (\sin x)/12$;
 $\sin x = 0.5456$; $x = 33$
 angle of elevation = $47 - 33 = 14$

23. B. Subtract the two equations:
 $x^2 - 2x + y^2 - 2y = 7$
 This is the equation of a circle.

24. A. $F(2)=7$; $F(3)=15$; $F(4)=31$; $F(5)=63$;
 $F(6)=127$; $F(7)=255$; $F(8)= 511$; $F(9)=1023$
 $F(10)=2047$

25. D. $6x = 30 + 360n$
 $x = 5 + 60n$; $x = 5, 65, 125, 185,$
 $245, 305$
 $x = 150 + 360n$
 $x = 25, 85, 145, 205, 265, 325$

26. C. Product: $\begin{vmatrix} 6 & 20 \\ 4 & 4 \end{vmatrix}$
 Determinant: $60 + 40 = 100$

27. D. $\tan 2x = \frac{2 \tan x}{1 - \tan^2 x} = \frac{3}{4}$
 $3 \tan^2 x + 8 \tan x - 3 = 0$
 $(3 \tan x - 1)(\tan x + 3) = 0$; $\tan x = 1/3$
 $\sin x = 1/\sqrt{10}$

28. C. Two asymptotes, one vertical and one slant

29. A. $\log \frac{3}{2} + \log \frac{4}{3} + \log \frac{5}{4} + \dots + \log \frac{2001}{2000} =$
 $\log \frac{2001}{2} = 3.00$

30. D. Sample space: H, TH, TTH, TTT

Let A = probability of three throws

Let B = probability of tails on first throw.

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{8} + \frac{1}{8}}{\frac{1}{4} + \frac{1}{8} + \frac{1}{8}} = \frac{1}{2}$