

# Trigonometry

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The abbreviation "NOTA" denotes "None Of These Answers."

1.  $-1570^\circ$  is coterminal with

- a)  $-50^\circ$       c)  $130^\circ$   
b)  $230^\circ$       d)  $190^\circ$       e) NOTA

2. What is the range and domain of  $y = \operatorname{arccsc} \phi$

- a)  $-1 \leq \phi \leq 1$       b)  $-1 \leq \phi \leq 1$       c)  $0 \leq y \leq \pi$       d)  $\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$       e) NOTA  
 $\frac{-\pi}{2} \leq y \leq \frac{\pi}{2}$        $0 \leq y \leq \pi$        $y \neq \frac{\pi}{2}$        $y \neq 0$

3. If a Tower is 146 feet high and makes an angle of depression with the sun of  $30^\circ$  how long is the Tower's shadow?

- a) 146 ft.      c) 292 ft.  
b)  $146\sqrt{2}$  ft.      d)  $292\sqrt{3}$  ft.      e) NOTA

4. Simplify in  $a + bi$  form:  $i^{243}(1-3i)(1+3i)$

- a)  $-10i$       b)  $-6 + 8i$       c)  $8i$       d)  $10$

5. Simplify: 
$$\frac{\frac{\sec x}{\tan x} - \csc x \cos^2 x + \cot x - \frac{\cos^3 x}{\sin x}}{\sin^2 x}$$

- a)  $\sin x$       c)  $\cos^2 x$   
b)  $\csc x + \cot x$       d)  $\sec x \tan x$       e) NOTA

6. Find the interior angle,  $a$ , at any vertex between two sides of a 15-gon. What is  $a \pmod{7}$ ?

- a) 0      b) 2      c) 4      d) 5      e) NOTA



13. Assume  $\Gamma_1 = \alpha - \beta$  and  $\Gamma_2 = \alpha + \beta$ , find  $\cos(\Gamma_1) + \cos(\Gamma_2)$ :

- a)  $2 \cos \alpha \sin \beta$       c)  $\cos(2\alpha\beta)$   
 b)  $2 \sin \alpha \sin \beta$       d)  $2 \cos \alpha \cos \beta$       e) NOTA

14. Given that  $\cos^2(2x) = p^2$  and  $\cos x + \sin x = \frac{1}{6}$ , find the value of  $p^2$ ?

- a)  $\frac{1}{36}$       b)  $\frac{71}{1296}$       c)  $\frac{1225}{1296}$       d)  $\frac{35}{36}$       e) NOTA

15. At the hike of the ball, I Stiffler, located at  $(0, 5)$ , runs at a speed of  $2\sqrt{2}$  units/sec on a bearing of  $315^\circ$  towards the QB for  $4 \sin\left(\frac{11\pi}{6}\right)$  seconds. Meanwhile, the QB has run to  $(0, -4\sqrt{3}+1)$ . On what bearing must I Stiffler run to sack the QB who is stopped looking for an open receiver?

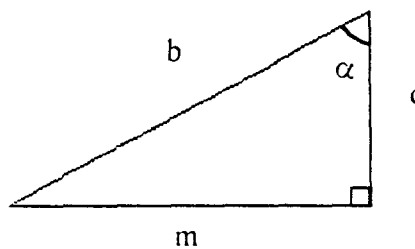
- a)  $60^\circ$       c)  $210^\circ$   
 b)  $120^\circ$       d)  $240^\circ$       e) NOTA

16. Given  $\sinh x = \frac{e^x - e^{-x}}{2}$  and  $\cosh x = \frac{e^x + e^{-x}}{2}$ ,  $\tanh(3 \ln 2) =$

- a)  $\frac{-35}{37}$       b)  $\frac{40}{41}$       c)  $\frac{63}{65}$       d)  $\frac{24}{25}$       e) NOTA

17. Find  $\sin \frac{\alpha}{2}$

- a)  $\frac{m}{b}$       c)  $\sqrt{\frac{b-c}{2b}}$   
 b)  $\sqrt{\frac{c-b}{2b}}$       d)  $\frac{m}{2b}$       e) NOTA



18. What is the sum of all solutions in the interval  $[0, 2\pi]$ :

$$2 \tan^2 x + \sec^2 x + 4 \sec x - 2 = 0$$

- a)  $4\pi$       b)  $\frac{3\pi}{2}$       c)  $\pi$       d)  $0$       e) NOTA

19. Dr. Carlen is flying to a math conference along latitude  $60^\circ$  North. What distance does he travel if he moves from longitude  $84^\circ$  W to  $16^\circ$  E via the Prime Meridian. (Assume that the radius of the earth is 6250 km)

- a) 1750 km                      c) 3500 km  
 b)  $\frac{5209\sqrt{3}\pi}{3}$  km              d)  $\frac{15625}{9}$  km                      e) NOTA

20. Each edge of the regular hexagon RMFISH is  $\frac{\sqrt{7}}{9}$  units long. What is the length of diagonal  $\overline{RS}$ ?

- a)  $\frac{\sqrt{21}}{3}$                       b)  $\frac{\sqrt{7}}{81}$                       c)  $\frac{\sqrt{7}}{3}$                       d)  $\frac{2\sqrt{21}}{9}$                       e) NOTA

21. Which of the following are true?

I.  $\arcsin^2 x + \arccos^2 x = 1$

II.  $\arctan(3x) + \arctan(3y) = \arctan \frac{3(x+y)}{1-9xy}$ ,  $xy \neq \frac{1}{9}$

III.  $\arcsin(2x) = 2 \arcsin x \arccos x$

- a) I only                                      c) I, II only  
 b) III only                                      d) I, II, III only                                      e) NOTA

22. How many of the following expressions can be shown to be equivalent to  $\tan^2 \theta$ ?

$$\frac{1+\cos(2\theta)}{1-\cos(2\theta)} \quad \frac{\sin^2 \theta}{\cos^2 \theta} \quad \frac{\cos^2 \theta - \cos^4 \theta}{\cos^4 \theta} \quad \frac{1+\cos(2\theta)}{2\cos^2 \theta}$$

- a) One                      b) Two                      c) Three                      d) Four

23. Simplify the expression  $\frac{a-bi}{b+ai}$ :

- a)  $-i$                                       c)  $i$   
 b)  $\frac{a^2-b^2+2abi}{a^2+b^2}$                       d)  $1+\frac{2abi}{a^2+b^2}$                       e) NOTA

24. The graph of the polar equation  $r=4+4 \sin \theta$  is which of the following:  
 a) Lemniscate b) Limacon c) Cardioid d) Dimpled limacon e) NOTA
25. If  $\sin \theta=5 \cos \theta$  on  $(0 \leq \theta \leq \frac{\pi}{2})$ , then what is the value of  $\sin(2\theta)$ ?  
 a)  $\frac{\sqrt{26}}{26}$  b)  $\frac{5}{13}$  c)  $\frac{5\sqrt{26}}{26}$  d)  $\sin(\frac{1}{2} \arccos \frac{1}{7})$  e) NOTA
26. Find the angular velocity of a circle if the linear velocity is  $\frac{2\pi}{3} \text{ cm/sec}$  at a point .25 m away from the center.  
 a)  $\frac{2\pi}{75} \text{ deg/sec}$  b)  $\frac{12}{5} \text{ deg/sec}$  c)  $\frac{8\pi}{3} \text{ deg/sec}$  d)  $\frac{24}{5} \text{ deg/sec}$  e) NOTA
27.  $\prod_{i=1}^{2000} \left( \frac{\sin\left(\frac{\pi(2i+1)}{4}\right)}{\cos\left(\frac{\pi(2i+5)}{4}\right)} \right) = ?$   
 a) 1 b) -i c) 0 d) x  
 e) NOTA
28. Solve:  $\arctan \frac{1}{2} + \arctan \frac{1}{3} =$   
 a) 0 b)  $\frac{\pi}{4}$  c)  $\frac{\pi}{3}$  d)  $\frac{\pi}{2}$  e) NOTA
29. Solve:  $\sin \left[ 2 \operatorname{Arccos} \left( \frac{3}{4} \right) \right] =$   
 a)  $\frac{3\sqrt{7}}{16}$  b)  $\frac{6\sqrt{43}}{43}$  c)  $\frac{6\sqrt{7}}{16}$  d)  $\frac{6}{5}$  e) NOTA
30. What is the area enclosed by the parametric equations:  
 $x = \frac{7}{2} - \frac{4}{5} \cos \theta$        $y = 3 - \frac{15}{8} \sin \theta$   
 a)  $\frac{3\pi}{2}$  b)  $\frac{13}{2}$  c)  $\pi^2$  d)  $\frac{21}{2}$  e) NOTA