

Gemini
FAMAT State Convention 2000

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Answer choice E is "NOTA" for "None of these answers."

1. What is the best approximation for the perimeter of the ellipse with equation

$$4y^2 + 9x^2 = 36?$$

A) $\pi\sqrt{26}$

B) 12π

C) 6π

D) $\pi\sqrt{22}$

E) *NOTA*

2. Using Descartes Rule for the equation

$$3x^7 - 4x^6 + 2x^5 + 3x^3 - x^2 - x + 1$$

which of the following can be determined?

- A. It has either 4 or 2 positive real roots.
- B. It has at least 2 imaginary roots
- C. It has either 1 or 3 negative real roots
- D. It has a triple root.
- E. *NOTA*

3. In how many zeros does the number $401!$ end?

- A. 95
- B. 99
- C. 100
- D. 101
- E. *NOTA*

4. What is the largest integer that cannot be equal to $26k + 15j$ where k and j are both whole numbers?

- A. 391
- B. 349
- C. 340
- D. 431
- E. *NOTA*

5. Who introduced the Arabic number system to Europe?

- A. Michael Servetus
- B. Nicholas Copernicus
- C. Zeno of Elea
- D. Leonardo da Pisa
- E. NOTA

6. Genma, a circus panda, wants some hot tea. He is located at (0,12) and the stove is located at (15,8). However, he must stop at the river, which flows along the line $y=0$, to fill up his teapot before he may use the stove. What is the shortest total distance he can travel if he goes from where he is to the river and then to the stove?

- A. 12
- B. 29
- C. 25
- D. 21
- E. NOTA

7. Brad, Travis and Phil are playing a game of paintball. In this game, they take turns shooting at each other. On each player's turn he may shoot at only one of the other two people, of his choice. Whenever someone is hit, they are eliminated and their turn is skipped from then on. First Travis shoots, then Phil, then Brad. If, at the end of a round, there is more than one person who is not eliminated, the people who are still not eliminated play another round, taking turns in the same order as before. The winner is the person who is not eliminated. The probability of Travis getting a hit on any given shot is 0.7. The probability of Brad getting a hit on any given shot is 0.3. Phil is in the US Marine Corps, so he never misses. Assuming that all players are playing to win and choose whom to shoot at accordingly, what is the probability that Brad will win? Assume that players may not intentionally miss. Round your answer to the nearest thousandth.

- A. 0.231
- B. 0.300
- C. 0.356
- D. 0.379
- E. NOTA

8. What is the area under the graph of

$$4x \cos^5 x - 3x \cos^3 x - x(\cos^2 x)(\cos 3x) = y$$

from $x=1$ to $x=e^2$?

- A. 1
- B. e^{-2}
- C. e^{-8}
- D. e^{-16}
- E. NOTA

9. Which of the following is equal to $\cos 5x$?

A) $\cos^5 x - 10(\cos^3 x)(\sin^2 x) + 5(\cos x)(\sin^4 x)$

B) $\cos^5 x - 5(\cos^4 x)(\sin x) + 10(\cos^3 x)(\sin^2 x) - 10(\cos^2 x)(\sin^3 x) + 5(\cos x)(\sin^4 x) - \sin^5 x$

C) $5(\cos^4 x)(\sin x) - 10(\cos^2 x)(\sin^3 x) + \sin^5 x$

D) $\cos^3 x - \sin^2 x$

E) *NOTA*

10. There are ten slots numbered 1-10 and ten blocks numbered 1-10 which fit into the slots. In how many ways can the blocks be placed into the slots so that no block is in its corresponding slot?

A. 3,628,799

B. 1,779,322

C. 579,401

D. 1,334,961

E. *NOTA*

11. For $\triangle ABC$, with sides a , b , and c opposite angles A , B , and C respectively, which of the following is the Law of Tangents?

A) $\frac{\tan(A+B)}{\tan(A-B)} = \frac{(\tan A + \tan B)(1 + \tan A \tan B)}{(\tan A - \tan B)(1 - \tan A \tan B)}$

B) $\frac{a-b}{a+b} = \frac{\tan \frac{A-B}{2}}{\tan \frac{A+B}{2}}$

C) $\frac{a}{b} = \tan A$

D) $\tan C = \frac{2abc \sin A}{a^2 + b^2 - c^2}$

E) *NOTA*

12. $\triangle QRS$ has points A , B , and C on line segments SQ , RS , and QR respectively. Line segments SC and AR intersect at a point T in the interior of the triangle. Given that the ratio of line segments SA to AQ is 3:1 and that the ratio of line segments SB to BR is 5:2, find $AT:TR$.

- A. 11:5
- B. 5:8
- C. 15:8
- D. 1:2
- E. NOTA

13. The medians of isosceles triangle $\triangle ABC$ intersect at D . Four circles are drawn centered around points A , B , C , and D such that each circle is tangent to the other three. If the radii of the three larger circles are 4, 4, and 8, what is the radius of the smaller circle?

- A. 1
- B. 1.5
- C. 0.75
- D. 0.5
- E. NOTA

14. Which of the following sets of vectors spans \mathfrak{R}^3 ?

A) $(5i + j + 4k), (3i - j + 2k), (2i + 2j + 2k)$

B) $(-i + \frac{1}{2}j + 3k), (-2i - j - k), (-4i + 5k)$

C) $(7i + 2k), (i - 4j - k), (4i + 12j + 5k)$

D) $(2i + j + 4k), (i - 2j + k), (2i + j + 3k)$

E) NOTA

15. There are 32 people in a class who have brown hair, 13 that have blue hair, 14 who have black socks, and 41 who have white socks. 12 of the people with brown hair have black socks and 2 of the people with blue hair have black socks. What is the probability that a person selected at random from the class who is found to be wearing black socks has blue hair?

A) $\frac{1}{7}$

B) $\frac{2}{55}$

C) $\frac{6}{7}$

D) $\frac{14}{55}$

E) NOTA

16. What Greek mathematician found the volume of a sphere by determining that the sphere was two-thirds the volume of the smallest cylinder that would contain it? (Hint: He had a cylinder and a sphere carved on his tombstone)

- A. Aristarcus of Samos
- B. Archimedes of Syracuse
- C. Oenopides of Chios
- D. Theophrastus of Eresus
- E. NOTA

17. Sean always writes incorrect disputes. However, Sean also writes very confusing disputes, so sometimes his disputes are accepted accidentally. If at any point in time after Sean turns in his dispute there are two people in the dispute center then the probability of his dispute being accepted is 0.25. If there are never two people in the dispute center after Sean turns in his dispute, the probability of it being accepted is 0.5. The dispute center is open for 30 minutes and there are only 2 people who work in the dispute center. Person A can arrive any time from 10 minutes to 20 minutes after the dispute center opens and will stay for only 5 minutes. Person B can arrive anytime between 5 minutes and 15 minutes after the dispute center opens and will stay for 10 minutes. Assuming that Sean turns in a dispute at the exact instant when the dispute center is opened, what is the probability that his dispute will be accepted?

A) $\frac{137}{288}$

B) $\frac{3}{7}$

C) $\frac{23}{48}$

D) $\frac{65}{144}$

E) *NOTA*

18. In how many ways can 8 sponsors sit around a circular table if two of the sponsors (Dr. Morris and Mrs. Hammer) cannot sit next to each other?

- A. 5040
- B. 3600
- C. 40,320
- D. 30,240
- E. NOTA

19. The oldest of the Seven Wonders of the Ancient World is the Great Pyramid of Giza. If it would have taken 40 years to build using slave labor, 35 years to build using dedicated volunteers, or 2 years to build using Alien technology, how many days would it take to build using all three (assume all years are 365 days long and round to the nearest whole day)?
- A. 640
 B. 789
 C. 659
 D. 908
 E. NOTA

20. Your friend, Effendi, is being commissioned to design the legendary Hanging Gardens of Babylon. One of the vines is going to be hung by attaching both of its ends to the ends of a beam. This particular species of vine hangs in a perfect parabola. The center of the beam from which the vine is hung must also be the exact focus of the parabola, by order of the King. If the beam is 16 ft. long and the vine is attached to the ends of the beam, what is the minimum height that Effendi must place the beam above the ground for the vine to hang properly (in other words, how far down will the vine hang)?

- A. 3 ft.
 B. 8 ft.
 C. 6 ft.
 D. 4 ft.
 E. NOTA

21. $|f(x)| + h(x) = \begin{cases} 2x & \text{if } x < -2 \\ 6x + 8 & \text{if } x \geq -2 \end{cases}$

Find $f(-8) + h(6)$.

- A. 12
 B. 0
 C. -4
 D. 16
 E. NOTA

22. Given these premises, which of the following cannot be concluded?

All dogs go to heaven.

At least one cat goes to heaven.

No males go to heaven.

Everything that is not male is female.

- A. All dogs are female.
 B. There exists a male cat.
 C. There exists a female cat.
 D. If a cat is male then it cannot be a dog.
 E. NOTA

23. Which of the following are tautologies?

- I. $(p \wedge q) \rightarrow (p \vee q)$
- II. $(p \vee q) \rightarrow (p \wedge q)$
- III. $\neg q \vee (p \rightarrow q)$
- IV. $q \rightarrow (\neg p \vee q)$

- A. II, III, & IV
- B. I & II
- C. I, III, & IV
- D. II & IV
- E. NOTA

24. Suppose $S(x)$ represents the predicate “ x is a friend of Socrates” and $D(x)$ represents the predicate “ x is a friend of Descartes”. Which of the following does NOT represent the proposition “No one is friend of both Descartes and Socrates”?

- A) $\forall x(\neg S(x) \vee D(x))$
- B) $\neg \exists x(S(x) \wedge D(x))$
- C) $\forall x \neg (S(x) \wedge D(x))$
- D) $\exists x(\neg S(x) \wedge \neg D(x))$
- E) *NOTA*

25. Atchar, who is piloting an American plane on a training mission, has run out of fuel and is now in free fall without a parachute. In a last ditch effort to save Atchar, his daredevil comrade, Ross, decides to throw his parachute out the window of his cockpit for Atchar to catch. Ross is flying along the parametric equations $x = -100t + 1200$, $y = 4000$ and the falling plane is following the path of $x = 50t - 600$, $y = -16t^2 + 5000$. Assuming the acceleration due to gravity is $-32 \frac{ft}{s^2}$ in the y direction and there is no air resistance, find the time, t , to three decimal places, at which Ross must drop the parachute in order to save Atchar. (Ross is letting the parachute go with no initial vertical velocity and its initial horizontal velocity will be equal to that of his plane at the time the parachute is dropped; All units are in feet and seconds).

- A. 2.972
- B. 2.104
- C. 3.841
- D. 3.206
- E. *NOTA*

26. a , b , and c are consecutive whole numbers with $a < b < c$. Given the following equations, find $\log_c ab^2$ to the nearest hundredth.

$$c^b + 620c^a = c^7$$

$$c^{22} + 620c^{21} = c^{c^2}$$

- A. 2.29
B. 3.98
C. 2.41
D. 1.79
E. NOTA
27. What is the graph of $z = 3y^2 + 6x^2$?
- A. Paraboloid
B. Hyperboloid
C. Cylinder
D. Sphere
E. NOTA
28. A hemisphere is cut by a plane that is parallel to the base of the hemisphere and 4 cm from the top of the hemisphere. If the circumference of the circle formed by the intersection of the hemisphere and the plane is 12π , what is the radius of the hemisphere?
- A. 8.5
B. 7
C. 6.5
D. 5.5
E. NOTA

29. The following program is written in c++.

```

class MAOStudent
{
private:
    int gpa;
    char* name;
    int invitationalRank;
    int regionalRank;
public:
    MAOStudent();
    ~MAOStudent() {delete name;} // I
    MAOStudent( MAOStudent& copyFrom const ) // II
        { gpa = copyFrom.gpa;
          invitationalRank = copyFrom.invitationalRank;
          regionalRank = copyFrom.regionalRank;
          *name = *copyFrom.name; } // III
    } // IV

```

Which numbered lines of code contain compile time errors?

- A. I & II only
 B. IV only
 C. III & IV only
 D. I, II, & III only
 E. NOTA
30. In the Olympics, a new sport, the “Javelin Catch”, is added. In this sport a runner takes off from the starting point at the same time that the javelin thrower throws his javelin from the same spot. The only way to score points is if the runner can catch the javelin. If the runner starts off at $t = 0$ running at a speed of $10 \frac{ft}{s}$ and the javelin thrower throws the javelin at an angle of 30° in the direction that the runner is running, how much initial velocity must he give the javelin in order for the runner to catch it? Assume that the runner never changes speed or direction and that the acceleration of gravity is $-32 \frac{ft}{s^2}$. Also assume that in order for the runner to catch the javelin he and the javelin must occupy the same point in space. Round your answer to the nearest thousandth.
- A. 11.333
 B. 11.547
 C. 20.000
 D. 17.321
 E. NOTA