Calculus Individual Test
FAMAT State Convention 2000

"NOTA" in each question denotes "None of the above"

1. By the Intermediate Value Theorem for Integrals: $\int_a^b f(x)dx = (b-a)f(c)$. For the case $f(x)=x$, c is:

a) $\frac{a-b}{2}$ b) $\frac{b-a}{2}$ c) $a+b$ d) $\frac{a+b}{2}$ e) NOTA

2. The area enclosed by the curve $y=\cos x$, where $0 \le x \le \frac{\pi}{2}$, the x-axis and the y-axis, is divided by the line x = c in the ratio of 3 to 2, the portion adjacent to the y-axis being the larger. Then c is:

a) $\sin^{-1}\frac{3}{5}$ b) $\sin^{-1}\frac{2}{5}$ c) $\sin^{-1}\frac{4}{5}$ d) $\sin^{-1}\frac{2}{3}$ e) NOTA

3. In approximating the root of an equation $f(x)=x^3+x-12$ by finding where the graph of $f(x)$ cuts the x-axis, it is found that $x=2.1$ is an approximate root. If the value of $f(2.1)$ is –0.639, then, using differentials, which of the following is best as a closer approximation?

a) 2.038 b) 2.102 c) 2.145 d) 2.188 e) NOTA

4. A function represented by a power series can:

a) not be differentiated in the interior of its interval of convergence
b) only have the first order derivative in the interior of its interval of convergence
c) be convergent for any order of derivative (1st deriv., 2nd deriv., etc...)
d) have all orders of derivatives in the interior of its interval of convergence
e) NOTA

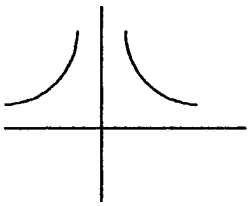
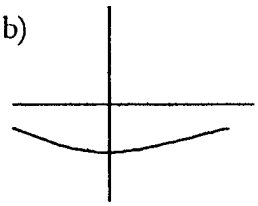
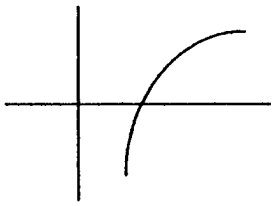
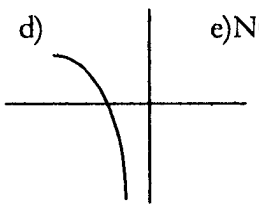
5. If $f(x)=\begin{cases} e^{-x^{-2}} & \text{if } x \neq 0 \\ 0 & \text{if } x=0 \end{cases}$ then the Maclaurin series represents $f(x)$ for:

a) all values of x b) $|x|<1$ only c) $|x|>1$ only d) $x=0$ only e) NOTA

6. The differential equation $\frac{dy}{dx}=y\left(\frac{1}{x}-1\right)$ for $x \geq 1$, represents a specific function with an inflection point at:

a) $x=-1$ b) $x=0$ c) $x=1$ d) $x=2$ e) NOTA

7. Which of the following curves best represents a possible solution to $xydx+dy=0$?

a)  b)  c)  d)  e)NOTA

8. When the area in square units of an expanding circle is increasing twice as fast as its radius in linear units, the radius is:

a) $\frac{1}{4\pi}$ b) $\frac{1}{4}$ c) $\frac{1}{\pi}$ d) 1 e)NOTA

9. The acceleration of a body moving in a straight line is given in terms of time t by a = 8 – 6t. If the velocity of the body is 25 at t = 1 and if s(t) is the distance of the body from the origin at time t , what is s(4) – s(2)?

a) 20 b) 24 c) 28 d) 32 e) NOTA

10. The length of the curve $y=\ln(\sec x)$ from $x=0$ to $x=b$, where $0<b<\frac{\pi}{2}$, may be expressed by which of the following integrals?

a) $\int_0^b \sec x dx$ b) $\int_0^b \sec^2 x dx$ c) $\int_0^b \sec x \tan x dx$ d) $\int_0^b \sqrt{1+(\ln(\sec x))^2}\,dx$ e) NOTA

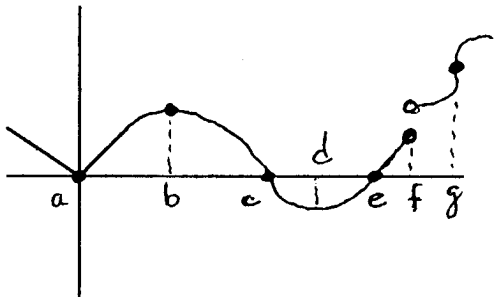
11. If the substitution $\sqrt{x}=\sin y$ is made in the integrand of $\int_0^{1/2} \frac{\sqrt{x}dx}{\sqrt{1-x}}$, the resulting integral is:

a) $\int_0^{1/2} \sin^2 y dy$ b) $2\int_0^{1/2} \frac{\sin^2 y dy}{\cos y}$ c) $2\int_0^{\pi/4} \sin^2 y dy$ d) $\int_0^{\pi/4} \sin^2 y dy$ e) NOTA

12. Use the graph to determine all x-values at which the function is not differentiable.

a) b & d b) a & f c) a, f & g

d) a, c & e e) NOTA



13. The position function for a particular object is $s = \frac{-29}{2}t^2 + 56t + 33$. Which statement is true?

a) The velocity is a constant. b) The velocity at time t = 1 is 74.5.

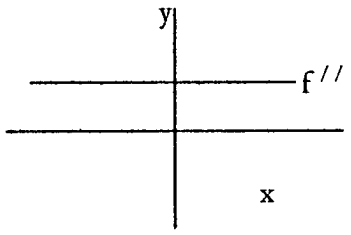
c) The initial position is 33. d) The initial velocity is –29. e) NOTA

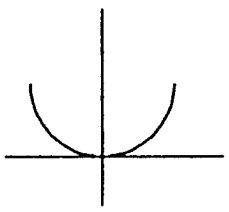
14. If it applies, use Rolle's Theorem for $f(x) = x^2 + 3x$ on [0, 2] to find all value(s) of c such that $f'(c) = 0$.

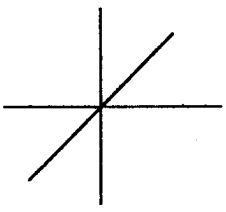
a) $c = \frac{-3}{2}$ b) $c = 0, -3$

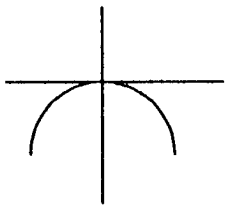
c) Rolle's Thm. does not apply because the function is not continuous on the interval.
d) Rolle's Thm. does not apply because $f(0) \neq f(2)$
e) NOTA

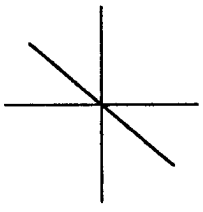
15. The figure given in the graph is the 2nd derivative of a polynomial function. Choose which graph could be the graph of the original function.



a) 

b) 

c) 

d) 

e) NOTA

16. Find the horizontal asymptote (if any) for $f(x)=\dfrac{ax^3}{b+cx+dx^2}$.

a) $y=\dfrac{a}{b}$ b) $y=0$ c) $y=\dfrac{a}{d}$ d) No horiz. asy. e) NOTA

17. Use differentials to approximate $\sqrt{3.3}$ to 3 decimal places (starting at x = 4).

a) 1.825 b) 1.817 c) 1.750 d) 1.650 e) NOTA

18. Evaluate: $\int_0^2 |x-1|\,dx$

a) 0 b) 1 c) ½ d) 2 e) NOTA

19. Use the Trapezoidal Rule , with n = 4, to approximate $\int_0^7 (x^2-7x)dx$ to two decimal places.

a) 57.17 b) 53.59 c) –57.17 d) –53.59 e) NOTA

20. Solve this differential equation: $2y' = y$.

a) $y=Ce^{x/2}$ b) $2y=\dfrac{y^2}{2}+C$ c) $y=e^{2x}+C$ d) $y=e^{x/2}+C$ e) NOTA

21. Evaluate: $\int \cos(\ln x)dx$

a) $\dfrac{x}{2}[\cos(\ln x)+\sin(\ln x)]+c$ b) 0 c) $\dfrac{x\ln x}{2}[\cos x+\sin x]+c$ d) $\dfrac{-\sin(\ln x)}{x}+c$ e) NOTA

22. Any function that can be differentiated twice must have at least how many points of inflection between any maximum and minimum?

a) 0 b) 1 c) 2 d) Cannot be determined e) NOTA

23. An arc of an ellipse with midpoint on the minor axis is revolved about the major axis to form the curved surface of a barrel shaped solid. Find the exact volume of the solid, given that its length is 6, the diameter of its midsection is 4 and the diameter of each of its ends is 2.

a) 9π b) 18π c) 27π d) 36π e) NOTA

24. A conical vessel is 12 feet across the top and 15 feet deep. If it contains a liquid weighing w lb/ft ^{3} to a depth of 10 feet, find the work done in pumping the liquid to a height 3 feet above the top of the vessel.

a) $230\pi w$ ft-lbs b) $560\pi w$ ft-lbs c) $1120\pi w$ ft-lbs d) $2240\pi w$ ft-lbs e) NOTA

25. A particle moves on the circle $x^2 + y^2 = 1$ so that at time $t \geq 0$ the position is given by the vector $\left(\frac{1-t^2}{1+t^2}, \frac{2t}{1+t^2}\right)$. Find the velocity vector.

a) $\left(\frac{2t}{(1+t^2)^2}, \frac{2}{(1+t^2)^2}\right)$ b) $\left(\frac{4t}{(1+t^2)^2}, \frac{2t^2}{(1+t^2)^2}\right)$ c) $\left(\frac{-4t}{(1+t^2)^2}, \frac{2-2t^2}{(1+t^2)^2}\right)$

d) $\left(\frac{4}{(1+t^2)^2}, \frac{2t^2}{(1+t^2)^2}\right)$ e) NOTA

26. Evaluate: $\lim_{x\to\infty} e^{-x}\ln x$

a) 0 b) ∞ c) 1 d) Does Not Exist e) NOTA

27. Evaluate: $\int_1^2 \frac{dx}{(2x-3)^{2/3}}$

a) 0 b) 3/2 c) 3 d) The integral diverges e) NOTA

28. What are the coordinates of the inflection point on the graph of $y=(x+1)Arc\tan x$?

a) (-1, 0) b) (0, 0) c) (0, 1) d) $(1, \frac{\pi}{4})$ e) NOTA

29. The speed of signaling through a submarine cable depends on the ratio (y) of the external diameter of the core to the diameter of the enclosed copper wire. Then the number of signals that can be sent per minute is given by: $S=(ay^2)\ln\left(\frac{1}{y}\right)$, where a is a constant. S is a maximum if:

a) $y=\sqrt{e}$ b) $y=\frac{a}{\sqrt{e}}$ c) $y=a\sqrt{e}$ d) $y=\frac{1}{\sqrt{e}}$ e) NOTA

30. $\sum_{n=0}^{\infty}\frac{(-1)^n x^n}{n!}$ is the Taylor's series about zero for which of the following functions?

a) sin x b) cos x c) e^x d) e^{-x} e) NOTA