**Solutions: Alpha Individual Test**
**Mu Alpha Theta State Convention 2000**



1. $\sin^2\theta + \cos^2\theta = 1$ so
$\cos\theta = \sqrt{1-(\frac{1}{3})} = \frac{2\sqrt{2}}{3} =$ D.

2. $(1+i)^2 = 1 + 2i - 1 = 2i$ so
$(1+i)^4 = (2i)^2 = 4i^2 = -4$. Choice D.

3. The triangle forms a 30-60-90 triangle with short leg BC, which is half the hypotenuse, or 5. Choice A.

4. $\cos^2 x + \frac{\cos^2 x}{\sin x} = \frac{\cos^2 x \sin x + \cos x}{\sin x}$
$= \frac{\cos^2 x(\sin x + 1)}{\sin x} = \cos^2 x(\frac{\sin x + 1}{\sin x})$
$= \cos^2 x(1 + \frac{1}{\sin x})$. So f(x) is equal to $1 + \csc x$, choice A.

5. E. None are true.

6. sinx can be equal to cosx in quadrants I and III, at a reference angle of $\frac{\pi}{4}$. Two answers, gives choice C.

7. Since $f(\csc x) = \sin x$ and
$f(\sin x) = \csc x$ and $f(\csc x) = \sin x$
then the answer is choice A.

8. The sum of the roots of the equation are $-\frac{b}{a}$ which is equal to 5. So
$5 = -3 + r$ and r=8. Choice C.

9. The sum of an infinite geometric series is given by $\frac{a_1}{1-r}$ which equals $\frac{\cos\theta}{1-\cos\theta}$. Setting this equal to 3/4 and solving gives $\cos\theta$ equals $\frac{3}{7}$ or choice B.

10. Let $x = \sqrt{6-\sqrt{6-\sqrt{6-\ldots}}}$ so that
$x = \sqrt{6-x}$ and $x^2 = 6 - x$ and $x^2 + x - 6 = 0$
which has two roots, 2 and -3. Since this is the positive square root, the answer is 2. Choice B.

11. Using Heron's formula, the area of the triangle is $\sqrt{s(s-a)(s-b)(s-c)}$ with s= the semiperimeter which is 7. So Area=
$\sqrt{7(7-3)(7-5)(7-6)} = \sqrt{54} = 2\sqrt{14}$.
Setting $\frac{1}{2}bh = 2\sqrt{14}$ and using the smallest side as the base, we get height = $\frac{4\sqrt{14}}{3}$
or choice A.

12. The sum of the focal radii of an ellipse is equal to 2a. Using $\frac{x^2}{4} + \frac{y^2}{9} = 36$ gives
$a = \sqrt{9} = 3$. So 2a=6 which is choice E.

13. $\frac{(\sqrt{2a}-\sqrt{a})(\sqrt{2a}-\sqrt{a})}{(\sqrt{2a}+\sqrt{a})(\sqrt{2a}-\sqrt{a})} = \frac{2a-2\sqrt{2a^2}+a}{2a-a} =$

$\frac{3a-2a\sqrt{2}}{a} = 3 - 2\sqrt{2} =$ choice A

14. $\frac{y^2}{4} - \frac{x^2}{2} = 1$ has asymptotes
$y = \pm\frac{\sqrt{4}}{\sqrt{2}}x$ or $y = \pm\sqrt{2}x$ so m $=\sqrt{2}$
or choice C.

15. Using similar triangles, and the fact that
$OD = OA = 1$ we get $\frac{\cos\theta}{1} = \frac{\sin\theta}{x}$ and
$x = \tan\theta$ which is choice B.

16. $(2\cos\theta - 3\sin\theta + 0) - (0 + 0 + 8\sin\theta) = 0$ or
$2\cos\theta - 11\sin\theta = 0$ which gives $\cos\theta = \frac{11}{2}\sin\theta$
which gives k=11/2 or choice C.

17. $2^{2x} = (2^{-3})^k$ so $2x = -3k$ and
$x = \frac{-3}{2}k$ which is choice C.

18. Rewriting as $3^8 \bullet 2^{18} \bullet 5^{20}$ and again as $3^8 \bullet 5^2 \bullet (2^{18}5^{18})$ we see the product has a factor of $10^{18}$ so k=18, choice D.

19. Using the triangle inequality theorem,
$2 < AC < 10$ and to make B obtuse we have $AC > \sqrt{4^2 + 6^2}$ so $AC > \sqrt{52}$.
The intersection: $\sqrt{52} < AC < 10$
so $x^2 + k^2 = 52 + 100 = 152$. Choice D

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20. The sum of the first 4 terms is 10 so the sum of the 84 terms is 21(10)=210 Choice B.

21. The circumference of the circle is $24\pi$ and the length of arc BD is $4\pi$ so we have the arc length is a portion of the circumference. $\frac{4\pi}{24\pi} = \frac{\theta}{2\pi}$ gives $\theta = \frac{\pi}{3}$ or choice A.

22. This is an SSA situation so angle B can be obtuse or acute. Using the law of sines: $\frac{\sin 32}{10} = \frac{\sin C}{12}$ gives the sinC to be $\frac{12\sin 32}{10}$ so $C \approx 39.4869992$ or the supplement of this angle. Therefore $B \approx 108.513$ or 7.4869992 Using the law of cosines: $144 + 100 - 2(12)(10)\cos B = AC^2$ gives AC could have length 17.89 or 2.46. Choice D could be a length AC.

23. g(x) is 9 for any value of x. Choice B.

24. The number of games played is C(5,2)=10. Each game is 100/10 = 10 minutes long. Each team will play four games, so each team will play 40 minutes which is choice D.

25. Putting the arch on the coordinate axes with vertex on (0,100) and the feet at ($\pm$40,0) gives equation of the parabola to be $y - 100 = -a(x-0)^2$. Substituting (40,0) we get a=$\frac{1}{16}$ and equation $y - 100 = -\frac{1}{16}x^2$. When y=6 we get x= $\pm$38.78 and the answer is choice C.

26. The roots of g are 5 and -3. Substituting each into f we see that f(-3)=0 so the root the two functions share is -3. Since -3 is a root, f(-3) and g(-3) is equal to 0 so we just get a sum of -3. Choice C.

27. The side DE is equal to $\sin\theta + \cos\theta$ so $(\sin\theta + \cos\theta)^2 = \frac{9}{8}$. Expanding gives $\sin^2\theta + 2\sin\theta\cos\theta = \cos^2\theta = \frac{9}{8}$. Since $\sin^2\theta + \cos^2\theta = 1$ we have $2\sin\theta\cos\theta = \frac{1}{8}$ and therefore $\sin 2\theta = \frac{1}{8}$. Choice B.

28. The stock's change is given by S(t) but we have no idea of the price of the stock. So choices B, C and D cannot be verified. Since cosine has a range of [-1,1] we see that this function is always positive. So the stock is always increasing in price. Choice A.

29. When Simone has $12,000, Juliet has two more years of interest. So the Juliet amount is (1.04)(1.04)(120000) = 12979 to the nearest dollar. Choice A.

30. The angle between the paths is 35 degrees. At 1:00 the boats will have traveled 4 mi and 3 miles respectively (d=rt). The distance between $A_1$ and $B_1$ is approximately 2.31092, obtained by the law of cosines. Angle $DA_1E$ is 45 deg. Using the law of Sines to find Angle $CA_1B_1$ we get approx. 48.125 degrees. Its supplement plus 45 degress gives $m\angle DA_1B_1$ which is approx 176.8746 degrees. Now using $\frac{1}{2}cbSinA$ to find the area we get $\frac{1}{2}(10)(2.31)(Sin176.8746)$ gives approx 0.6299 which rounds to choice A.
Note: although values in these solutions may be written in rounded form, memory storage of the calculator was used so that more significant digits were actually used to obtain the final answer stated here.